Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 212 OCTOBER, 1931 No. 4

RESEARCHES ON WOOD FIBERS AS A PAPER-MAKING MATERIAL.*

BY

GEORGE A. RICHTER,

Director of Research, Brown Company,
Berlin, N. H.

Modern means of communication are becoming increasingly dependent upon the use of paper as a medium upon which to inscribe records of acts, deeds, agreements and correspondence. While the use of telephone and radio messages has in many cases superseded written correspondence, all important transactions are ultimately confirmed and recorded by written word or by printed document.

The use of paper for the inscription of records, messages and the like dates back to the early ages. The choice of paper for this purpose rather than cloth or leather is very obvious when one considers the cost and the availability of raw material and the ease with which paper can be printed and bound into suitable form. There may be isolated instances which demand the use of fabric or of suitable tanned skins, but in general they are used only when special artistic or decorative effects are desired.

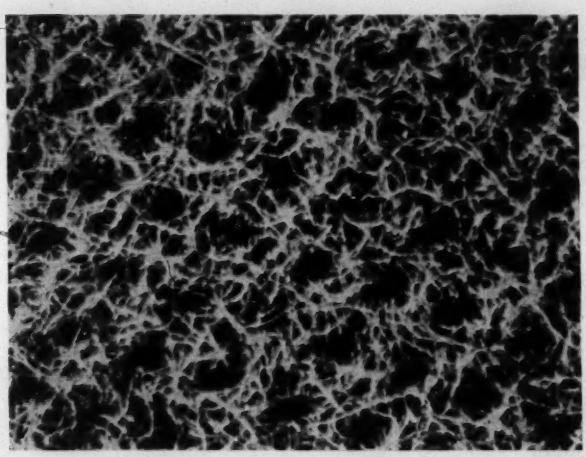
Paper may be defined as a sheet of cellulosic material in which the fiber units are distributed in a more or less random manner and in which the fibers are bound or cemented together

* Presented at a meeting held Thursday, January 29, 1931.

(Note.—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

by a cellulose gel. (See Plates I, II, and III.) It is true that there are special products in which the bond may comprise other cementing agents such as rubber or gelatin, but for the most part the individual fibers are bound by the cellulose gel which is formed when normal cellulose is rubbed or macerated in the presence of water.

PLATE I.



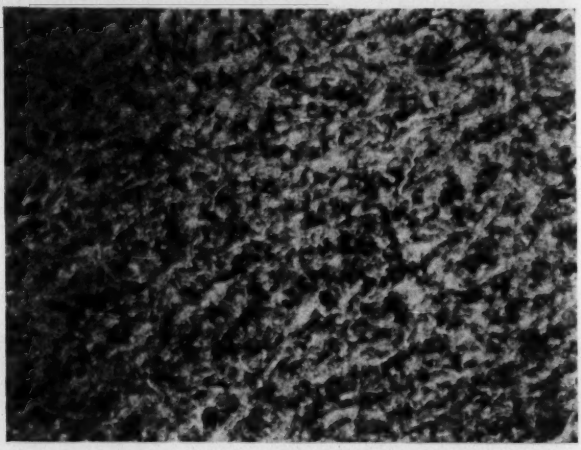
Magnified view of thin, loosely felted, fibrous cellulose sheet.

The fact that cellulose should prove to be peculiarly suited for the preparation of paper may be explained by its properties. Cellulose occurs in nature in the form of fibers which in themselves lend reinforcement to a sheet which may be constructed therefrom. The cellulose molecule possesses great affinity for water and when beaten in its presence will form the characteristic mucilaginous gel and fibrillæ, which will bind the longer fibers that are present into a continuous self-supporting sheet. (See Plates IV, V, and VI.) This sheet, when dry, possesses a mechanical strength which is largely dependent upon the quality and the percentage of the

gel that is present. Cellulose differs from wool, leather, and silk in this characteristic behavior in the presence of water.

There are literally hundreds of different types of paper, differing from each other in composition, texture, physical strength, and relative permanence. The vast range of types may be illustrated by comparison of a soft, bulky, absorbent blotting paper with a hard, tough, impervious sheet of parchment paper. There is an equally great contrast between

PLATE II.



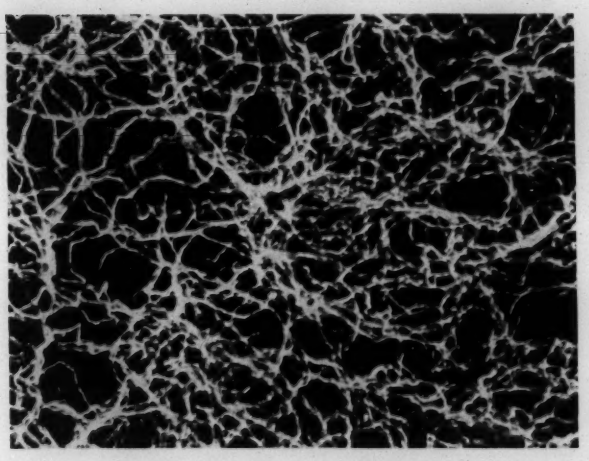
Surface magnification of thin manifold paper.

a well-sized ledger stock that has been produced from a high-quality rag fiber and which is intended for use in permanent records, and a weak sheet of groundwood newsprint which is lightly sized and very impermanent in character.

The desired variations in the character of papers may be controlled and determined in many ways. The following major factors play a part in the formation and properties of paper:

1. The origin and the type of cellulose fiber which is used to produce the paper.
2. The procedure that is employed to cut and hydrate the fiber.
3. The amount and kind of sizing agents, fillers and other extraneous materials that are used with the fiber.

PLATE III.



Loosely felted sheet of unbeaten wood fiber.

4. The mechanical contrivance on which the sheet is formed from the beaten and prepared fiber; i.e., Fourdrinier or cylinder machine.
5. The method by which the paper is dried.
6. After-treatments, such as the use of calenders, platers or chemical reagents.

It is evident that the means whereby a papermaker can produce the variety of papers that he desires are to a large extent within his control. The upper limit to his achievements, however, is mainly set by the first of the above factors;

each kind of fiber offers certain quite definite possibilities, which determine the class of paper that it is possible to make and the general procedure that the artisan must adopt to make it. It is beyond the scope of this paper to enumerate in detail the properties of each of the different cellulose stocks

PLATE IV.

Old rag stock hydrated in a hollander beater to an average papermaking condition.

which are used to make paper, but it will be instructive to cite a few examples illustrating the limitations that are imposed by the properties of the cellulose.

A cotton rag pulp that has been prepared by subjecting a new white rag stock to a mild alkaline digestion and a light bleach treatment is especially suited for conversion into a

permanent, tough, flexible ledger paper. On the other hand this specially prepared rag fiber would be useless for the production of a transparent, highly compact glassine sheet. The glassine papers are best prepared from a somewhat undercooked sulphite pulp—that is, one which is made by

PLATE V.

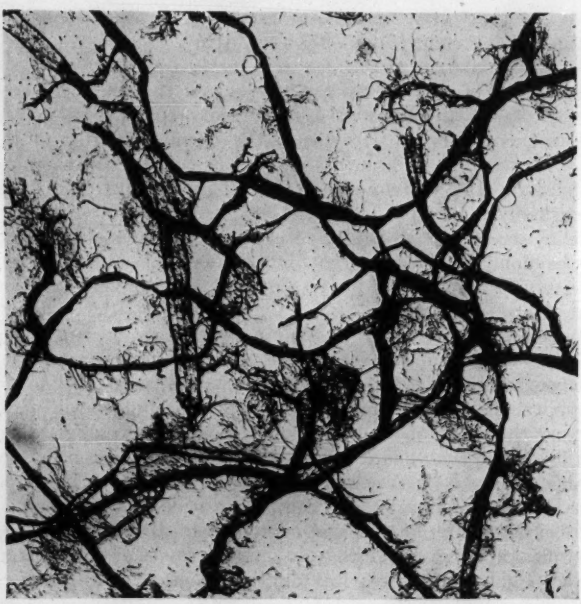
New rag fiber that has been hydrated in a hollander beater to a papermaking condition.

suitable digestion of wood with a bisulphite solution at elevated temperatures. On the other hand, sulphite pulp can be used as an ingredient in ledger paper, but because of the impurities which are present in the sulphite stock and because of the presence of hydrolyzed cellulose and oxidized products which are formed during the acid digestion and

subsequent bleaching treatment, the sulphite ledger sheet is appreciably weaker, more brittle, and much less permanent than is the rag-fiber paper.

Soda pulp and esparto pulp, which are prepared by the alkaline digestion of deciduous wood and esparto grass, respectively, consist of fibers that are quite short and which

PLATE VI.



A purified wood fiber that has been hydrated in a hollander for conversion into paper.

lend bulk, opaqueness, flatness and softness to a paper in which they are incorporated. These short, weak fibers are rarely used alone; they are usually mixed with longer, stronger fibers to furnish the needed strength. Neither the soda pulp nor the esparto fiber is as pure as a high-quality rag stock, and neither is considered in the same class for permanence.

Until quite recently it has been the experience of paper-makers, printers and publishers that the relative permanence of papers was closely allied with the rag-fiber content of the papers in question. There is considerable evidence that 100 per cent. rag papers that were made from 50 to 100 years ago have in general withstood the ravages of time, but that papers which were produced from other than cotton and linen stocks—i.e., sulphite pulps, groundwood, etc.—have perished in less than fifty years. In fact, some of the wood-fiber papers have endured for only a few years.

In seeking a reason for the marked difference in the durability of century-old rag papers and the sulphite wood-pulp papers, investigators have endeavored to correlate the permanence of the paper with the chemical purity of the fiber used. It is known, for instance, that the chemical characteristics of refined cotton and of sulphite pulp differ markedly in the percentage of so-called alpha- or resistant-cellulose. This alpha-cellulose may be defined as that ingredient of a lignin-free fiber which will withstand the dissolving action of an 18 per cent. sodium hydroxide solution at room temperature. As the term implies, the resistant residue is considered to be the most stable constituent of the fiber. In the absence of other substances which tend to decompose cellulose—e.g., acid, rosin, iron, and the like—it appears that a fiber that is high in alpha-cellulose will withstand the effect of weathering agents more successfully than will a fiber that has a lower percentage of alpha-cellulose. This finding is further confirmed by the fact that if cotton or linen undergoes drastic chemical treatments that result in a sacrifice of alpha-cellulose content, the fiber becomes less permanent, and paper that is made from such overtreated fiber is in many cases less durable than a good grade of sulphite paper.

If the resistant- or alpha-cellulose content of a pulp determines its potential life, then it would appear probable that when a wood cellulose is refined properly to eliminate non-alpha-cellulose constituents, the purified fiber should possess a correspondingly increased degree of permanence. Furthermore the physical behavior of the fiber might be changed by the elimination of the non-resistant cellulose

groups so that the fiber would more nearly approximate the papermaking qualities of cotton fiber. This conception has undoubtedly been shared by many investigators in the field of cellulose research.[1, 2] Attempts have been made to produce a cellulose that is high in alpha-cellulose from other raw materials than cotton and linen. In many cases such endeavors had as their objective the use of a suitable purified wood fiber for esterification purposes, rather than for conversion into paper. During the recent war period the investigations dealt mainly with the possible substitution of wood cellulose for cotton in the preparation of gun-cotton.[3] There is occasional evidence that some workers recognized the significance of a purified wood fiber in the paper industry. Until recently, however, there has been no record that a purified wood fiber was employed successfully as a stock for high-quality paper and it is very probable that the development of the purified wood-fiber industry awaited the perfection of procedures by which wood fiber can be purified without sacrificing the papermaking properties. To clarify this point, one need but mention that not all cellulose can be hydrated to form a gel and that inasmuch as the formation of a mucilaginous gel is necessary to bond the fibers together, if such hydration does not occur in the beater, there can be no coherent sheet of paper. A mercerized fiber may be high in alpha-cellulose but such a fiber will not hydrate satisfactorily and cannot be used successfully in ordinary papers. Similarly, it is common knowledge [4] that rayon, which is in reality a cellulose that has been precipitated from a viscose solution in the form of continuous filaments and which in the rayon process has undergone a distinct mercerization treatment, will not hydrate when beaten, and for this reason is objectionable in a rag stock that is intended for conversion to paper.

In the preparation of a refined wood fiber in which the alpha-cellulose has been materially increased and which is to serve as a base for conversion into paper, it is necessary to

[1] Hall, Gösta, *Paper Trade Journal*, **82**, 52 (April 8, 1926).
[2] Rasch, R. H., *Bur. Stds. J. Research*, **3**, 469 (1929).
[3] Edwardes, V. P., U. S. Pat. No. 1,310,694. July 22, 1919.
[4] Shaw and Bicking, *Paper Trade Journal*, **91**, 65–67 (Sept. 4, 1930).

adopt procedures which will effect an elimination of the non-alpha-cellulose constituents without injury to the papermaking properties of the purified residue.

This dual result has been achieved in recent years. By a proper relation of refining steps, it is now possible to produce a brilliantly white fiber in which the alpha-cellulose content will be maintained at from 94 to 96 per cent. and which will respond satisfactorily to such a hydration or beating action as has been found most suitable for rag halfstocks. The paper that can be produced from such a purified wood fiber simulates the soft, flexible sheet that characterizes a rag stock and it possesses permanence qualities of the highest degree. It differs from the usual wood fiber pulps in that—

(1) It cannot be readily transformed into a transparent glassine sheet.
(2) It will not hydrate rapidly.
(3) The resulting papers are distinctly superior in tearing resistance, folding endurance, and flexibility.
(4) It is far more durable than sulphite pulps.

As experimental data will show, the degree of purification and the accompanying differences in the physical properties can be varied widely to meet the requirements of the many grades of paper into which this purified wood fiber may be converted. It is possible, for instance, to exaggerate the softness in texture or to emphasize the folding endurance of the manufactured paper by suitable modifications in the pulp refining process. It is also possible to secure different degrees of chemical purity just as is the case when cotton rags are prepared for the beater. Uniformity of quality is believed to be easier to maintain with wood as a raw material than with cotton rags, since it is possible to obtain supplies of raw wood that are fairly constant in composition, while rags are variable both in a textile sense and in previous history.

In order to compare and appreciate more fully the differences and the similarities of the chemical and physical properties of the various fibers which are used in the paper industry, the following tables and curves have been compiled. Table I relates to the chemical tests only. The values that are cited are quite representative and indicate the position which is occupied by the purified wood fibers.

TABLE I.

Fiber.	Color.	Per Cent. Alpha Cellulose.	Per Cent. Ash.	Cuprammonium Solution Viscosity.	Copper Number.
Bleached Soda Pulp	Light cream	81.0	0.53	0.60	2.62
Esparto Pulp	Light cream	79.20	—	—	4.48
New Rag Stock Shirt Cuttings	White	97.9	0.17	11.7	1.14
Old Rag Half-Stock	Cream	92.9	1.3	3.5	1.9
Old Rag Half-Stock	Gray	75.5	0.53	0.40	4.6
Bleached Sulphite Pulp	White	84.2	0.22	3.0	3.5
Bleached Sulphite Pulp	White	82.6	0.35	4.0	4.2
Purified Wood Fiber No. 1	White	93.0	0.20	6.0	1.5
Purified Wood Fiber No. 2	White	94.5	0.15	8.0	1.2
Purified Wood Fiber No. 3	White	95.5	0.15	20.0	1.0
Purified Wood Fiber No. 4	White	96.0	0.15	12.0	1.4
Purified Wood Fiber No. 5	White	97.5	0.20	0.50	1.3

Note: Significance and procedures for alpha cellulose, ash, cuprammonium solution viscosity, and copper number are discussed in Industrial & Engineering Chemistry, **23**, 131 (February, 1931).

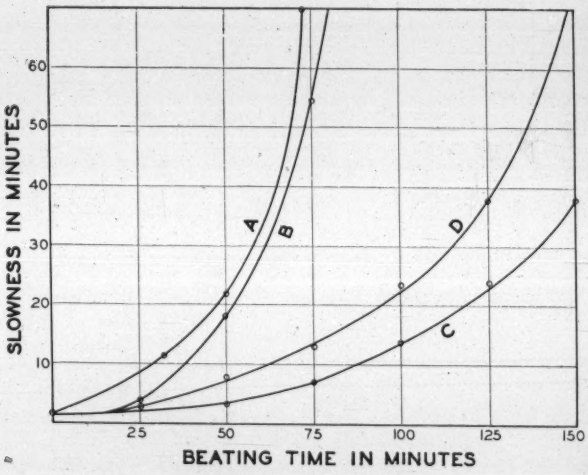
The data in Table I show clearly that there is no relationship between the source of the cellulose and the alpha-cellulose content of the fiber which has been prepared for the papermaking process. The values suggest that by proper elimination of extraneous matter, the alpha-cellulose percentage of a raw cellulosic material may be enhanced very appreciably, and that by severe chemical treatment a fiber that is originally high in alpha cellulose may suffer degradation.

HYDRATION OF CELLULOSE.

One can best illustrate the hydration properties of the various pulps by charting the changes that occur progressively as the representative stocks are subjected to continued beater action. This phase of the work has been investigated very fully, inasmuch as a high alpha-cellulose content signifies nothing to the papermaker unless the fiber in question lends itself successfully to the various papermaking steps. In this study the several fibers have been hydrated in many ways in order to establish relationships for various types of beating.

In this paper it will suffice to consider two general procedures: One involves the use of ball mills of the design ordinarily employed by modern laboratories [5] in which pulps are evaluated; the second consists of a beater or hollander type of machine. This hollander is very similar to that which is used in most paper mills. The results of the two different

PLATE VII.



Hydration curves of various wood pulps.

A = Poplar soda pulp of commercial grade.
B = Bleached sulphite pulp from softwood.
C = A purified wood fiber produced from spruce wood.
D = A second lot of purified wood fiber produced from spruce wood.

All beatings were made in porcelain ball mills. (See procedure, Wolf, R. B., Paper *19*, 11, Nov. 8, 1916.)

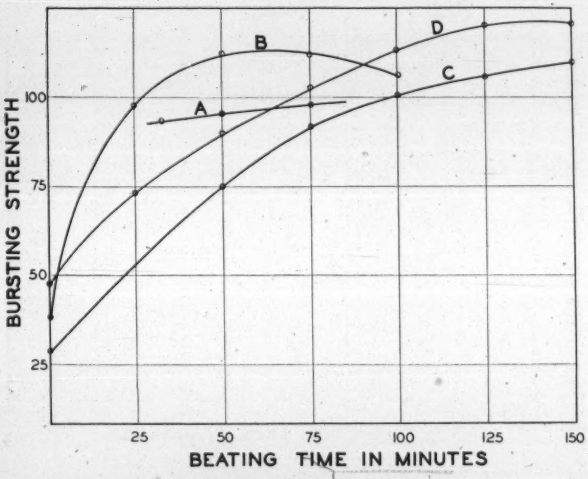
procedures were quite consistent, thus supporting the general belief that the data which are obtained with ball mills are of direct value in the prediction of results that may be expected in the hollander type of beater.

The tests were conducted in such a way as to allow a study of changes that occur as the beating is continued.

[5] Wolf, R. B., Paper, **19**, 11 (Nov. 8, 1916).

The degree of hydration is measured in units of "slowness," expressing the relative difficulty with which water drains from the beaten stock through a standard wire mesh screen, which retains the beaten fiber in the form of a mat when the fiber-water mixture is added thereto. The units of "slowness" are expressed in minutes of time that is required to allow a

PLATE VIII.



Bursting strength curves of various wood pulps.

A = Poplar soda pulp of commercial grade.
B = Bleached sulphite pulp from softwood.
C = A purified wood fiber produced from spruce wood.
D = A second lot of purified wood fiber produced from spruce wood.

All beatings were made in porcelain ball mills. (See procedure, Wolf, R. B., Paper *19*, 11, Nov. 8, 1916.)

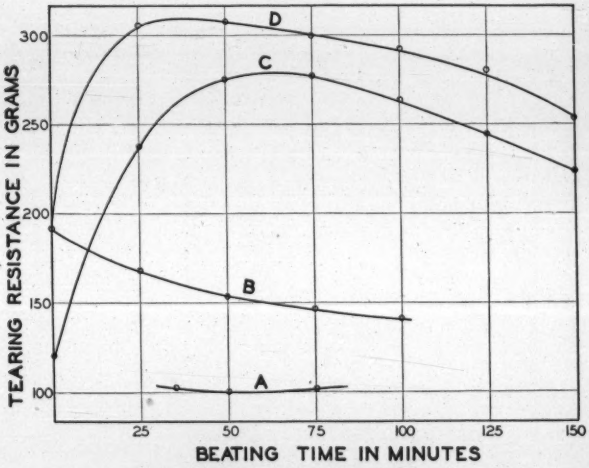
standard volume of water to drain through the mat as it is formed. Increased severity of hydration of fiber results in a product of higher "slowness," although various types of cellulose respond differently in the ease with which they can be hydrated.[6] The successive samples of fiber which were taken as the beating continued were converted into paper

[6] Richter, G. A., *Ind. Eng. Chem.*, 23, 266 (March, 1931).

and the paper was then tested under controlled humidity conditions [7] for bursting or "pop" strength, tearing resistance, and folding endurance.[8, 9, 10, 11]

The curves in Plates VII, VIII, and IX were obtained by charting the data from a series of ball mill beatings. Each

PLATE IX.



Tear resistance curves of various wood pulps.

A = Poplar soda pulp of commercial grade.
B = Bleached sulphite pulp from softwood.
C = A purified wood fiber produced from spruce wood.
D = A second lot of purified wood fiber produced from spruce wood.

All beatings were made in porcelain ball mills. (See procedure, Wolf, R. B., Paper 19, 11. Nov. 8, 1916.)

of the respective pulps was processed in exactly the same manner and hydrated for increasing periods of time as noted on the charts. It is quite evident that the poplar soda pulp

[7] All physical tests cited in this article were made at 45 per cent. relative humidity, and 70° F.

[8] Bureau Standards Circular No. 107 (1921).

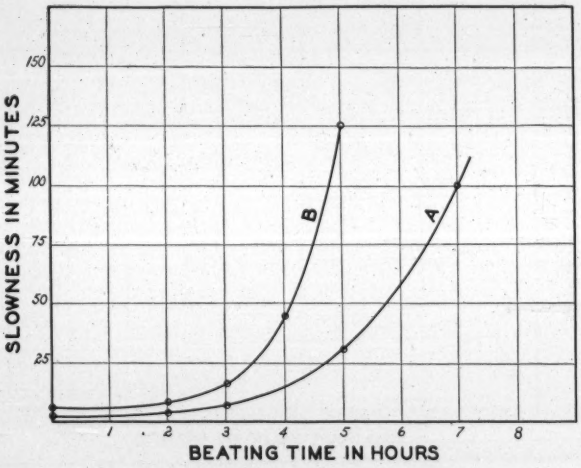
[9] Richter, G. A., *Ind. Eng. Chem.*, **23**, 131 (February, 1931).

[10] Richter, G. A., *Ind. Eng. Chem.*, **23**, 266 (March, 1931).

[11] Richter, G. A., *Ind. Eng. Chem.*, **23**, 371 (April, 1931).

and the softwood sulphite stock hydrate more quickly than do the two lots of purified wood fiber. Likewise, it requires longer beating time to develop the maximum bursting strength of paper from purified wood fiber. The tearing resistance of the resulting purified wood fiber papers is decidedly higher than either of the two other wood pulps. The folding endurance values of these papers were not included in this series of experiments, but appear in the later comparisons of hollander-

PLATE X.



Hollander hydration curves of sulphite pulp and purified wood fiber.
A = A commercial grade of white purified wood fiber that was produced from spruce wood.
B = A typical bleached sulphite pulp that was produced commercially from softwood.

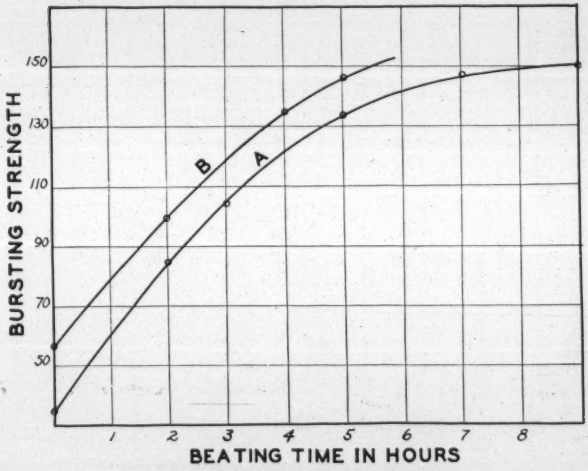
beaten stocks. It is to be noted that all of the above tests were made in the absence of sizing agents and alum. They, therefore, portray inherent differences in the behavior of the different pulps.

The results of the hollander beatings are shown graphically in Plates X, XI, XII, and XIII. In this set of experiments, commercial grades of sulphite softwood pulp and purified softwood fiber were hydrated under conditions as nearly

identical as possible. Samples were removed from the beater at intervals and were then converted into handmade paper and tested.

Plate X shows quite clearly that the high alpha-cellulose fiber resists hydration more than does the sulphite pulp. From Plate XI it would appear that the sulphite pulp will produce a paper of somewhat higher "pop" test, although

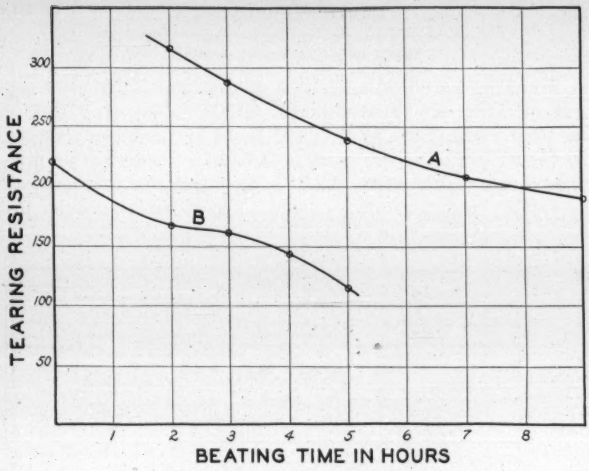
PLATE XI.



Bursting strength curves of pulps beaten in hollander.
A = A commercial grade of white purified wood fiber that was produced from spruce wood.
B = A typical bleached sulphite pulp that was produced commercially from softwood.

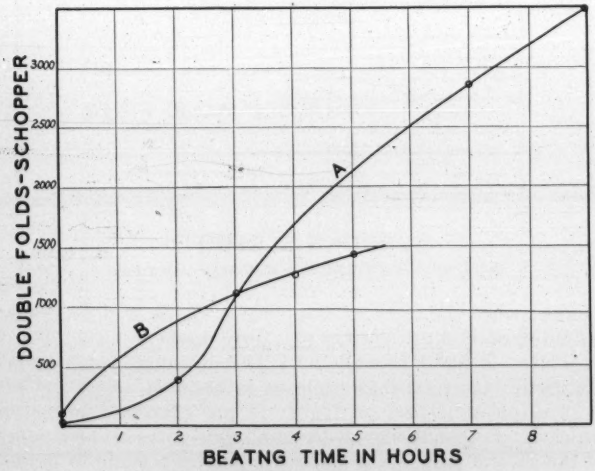
this relationship does not exist when the respective beaten stocks are converted into paper on a fourdrinier paper machine. The usual restraint in shrinkage of sheet when it is dried on dryer rolls apparently results in greater sacrifice in "pop" test than in the case with the purified fiber. Plate XII and Plate XIII show the decided superiority in tearing resistance and folding endurance of the papers that are formed from the hydrated fiber of higher alpha-cellulose content.

PLATE XII.



Tear resistance curves of pulps beaten in hollander.
A = A commercial grade of white purified wood fiber that was produced from spruce wood.
B = A typical bleached sulphite pulp that was produced commercially from softwood.

PLATE XIII.

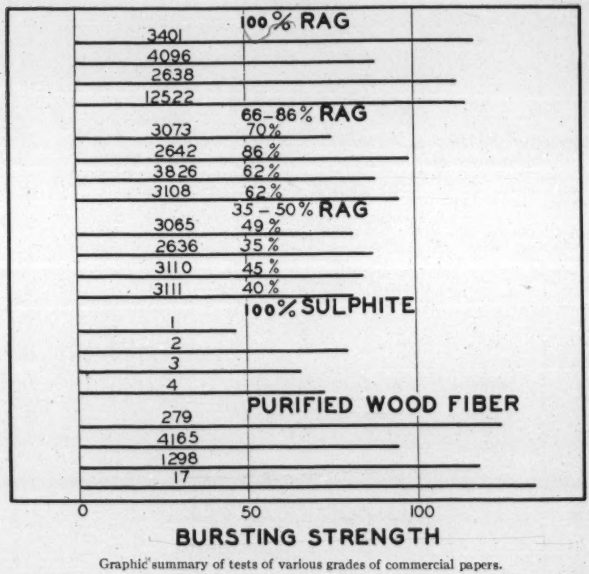


Folding strength curves of pulps beaten in hollander.
A = A commercial grade of white purified wood fiber that was produced from spruce wood.
B = A typical bleached sulphite pulp that was produced commercially from softwood.

TESTS OF COMMERCIAL PAPERS.

The above studies have been supplemented by tests that were made on several commercial grades of paper. Most of the papers examined were of the bond and ledger types and the higher grade papers were glue-sized. Inasmuch as glue-sizing tends to decrease the tearing resistance and increase the folding endurance of paper, it is necessary to consider this

PLATE XIV.



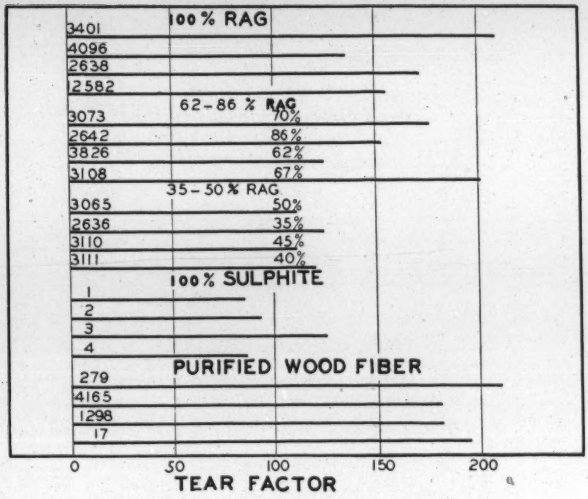
Graphic summary of tests of various grades of commercial papers.

element when the tests are compared. Table II summarizes a limited portion of the tests that were made.

Plates XIV, XV, and XVI are given to supplement graphically the tests that are listed in Table II.

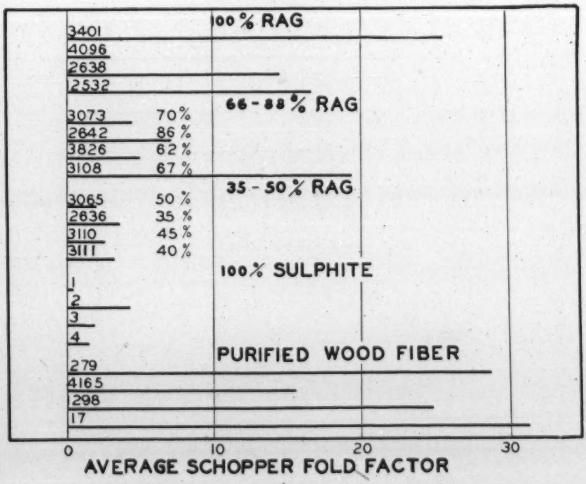
The physical tests of Table II are expressed in units which automatically correct for small differences in the weight of the several papers. No attempt has been made to grade

PLATE XV.



Graphic summary of tests of various grades of commercial papers.

PLATE XVI.



Graphic summary of tests of various grades of commercial papers.

TABLE II.

Tests of 100 Per Cent. Rag Papers.
All Glue-Sized.

Sample No.	Fiber Furnish.	Basis Weight.	Folio Weight.	Strength.	Tear Factor.		Folding Endurance.		Fold Factor.		Sizing.			Formation.	Watermark.
				% Points /Pound.	With.	Across.	With.	Across.	With.	Across.	Glue.	Rosin.	Starch.		
3401	100% Rag	53.0	23.8	118	205	213	1205	1512	22.7	28.5					
3402	100% "	53.0	23.8	117	172	183	1392	1473	26.2	27.7					
4096	100% "	51.7	23.5	89	131	139	112	182	2.1	3.5					
2246	100% "	55.8	25.1	96	183	196	659	1169	11.8	20.9					
2639	100% "	46.0	20.7	119	213	181	2327	1370	50.7	29.7					
2067	100% "	46.8	21.0	131	199	171	792	1011	16.9	21.6					
3069	100% "	46.6	21.0	110	214	214	907	1124	19.4	24.1					
2638	100% "	44.6	20.1	113	174	169	541	746	12.1	16.7					
3405	100% "	45.7	20.6	138	174	200	1069	2190	23.3	47.9					
3827	100% "	44.0	19.8	104	161	173	311	631	7.0	14.3					
12359	100% "	43.2	19.5	114	152	171	893	661	20.6	15.3					
473	100% "	43.0	19.4	92	159	144	532	513	12.3	11.9	7.42%	1.15%		10	10
474	100% "	45.0	20.2	105	136	174	788	1060	17.5	23.5	7.22%	1.81%		3	7
12360	100% "	44.4	20.0	122	164	191	760	1022	17.1	23.0					
12361	100% "	45.4	20.4	122	198	195	1654	1584	36.4	34.9					
12460	100% "	42.1	19.0	129	182	205	1278	1187	30.3	28.1					
12532	100% "	44.2	19.9	115	147	164	657	808	14.8	18.2		2.54%			
10403	100% "	44.1	19.8	131	193	211	1003	1147	22.7	26.0					
10341	100% "	55.7	25.1	92	155	166	245	392	4.5	7.0	3.93%				
10342	100% "	53.8	24.2	104	138	138	205	156	3.8	2.8	4.36%				
9706	100% "	54.8	24.7	118	203	214	1940	1211	35.4	22.0	3.65%				

TABLE II—(*Continued*).

Tests of 75 ± Per Cent. Rag Papers.
All Glue-Sized.

Sample No.	Fiber Furnish.	Basis Weight.	Folio Weight.	Strength.	Tear Factor.		Folding Endurance.		Fold Factor.		Sizing.			Formation.	Watermark.
				% Points /Pound.	With.	Across.	With.	Across.	With.	Across.	Glue.	Rosin.	Starch.		
3073	70% Rag	66.0	29.7	76	169	185	249	330	3.7	5.0					
2642	86% "	53.0	23.9	99	152	155	357	392	6.7	7.4					
2637	83% "	53.8	24.2	101	174	191	666	1198	12.3	22.2					
3830	68% "	54.8	24.7	95	146	166	421	419	7.6	7.6					
3826	62% "	44.9	20.2	89	117	134	146	302	3.2	6.7					
5581	70% "	45.7	20.6	94	126	154	295	419	6.4	9.1					
5583	73% "	42.4	19.1	101	137	162	535	623	12.6	14.6					
2248	78% "	49.8	22.4	114	154	157	445	799	8.9	16.0					
2245	66% "	36.3	16.4	63	77	83	18	17	0.4	0.4					
3108	67% "	46.0	20.7	96	202	200	785	979	17.2	21.5					
	Tests of 50 ± Per Cent. Rag Papers. *All Glue-Sized.*														
3065	49% Rag	42.6	19.2	82	104	117	63	102	1.4	2.3					
3066	42% "	42.5	19.2	87	116	118	130	136	3.0	3.2					
3068	50% "	43.8	19.7	91	126	137	356	424	8.1	9.6					
5582	47% "	45.2	20.4	94	118	143	582	484	12.8	10.7					
2636	35% "	45.4	20.4	88	122	128	110	250	2.4	5.5					
3110	45% "	45.0	20.2	85	103	120	134	92	3.0	2.0					
3111	40% "	45.0	20.2	82	111	129	116	160	2.6	3.6					

TABLE II—(*Continued*).

Tests of Purified Wood Fiber Papers.

Sample No.	Fiber Furnish.	Basis Weight.	Folio Weight.	Strength.	Tear Factor.		Folding Endurance.		Fold Factor.		Sizing.			Formation.	Watermark.
				% Points / Pound.	With.	Across.	With.	Across.	With.	Across.	Glue.	Rosin.	Starch.		
279	100% P. W. F.	47.6	21.4	125	204	219	1150	1604	24.1	33.6	Yes	Yes		6	8
4165	100% "	56.4	25.4	95	183	181	980	785	17.3	13.9	Yes	Yes			
4821	100% "	64.5	29.0	105	186	132	1501	606	23.3	9.4	Yes	Yes			
14	100% "	47.4	21.4	116	141	167	644	1043	13.5	22.0	Yes	Yes		4.5	5
17	100% "	48.4	21.8	135	187	202	1247	1824	25.7	37.6	Yes	Yes		5.5	7
372	100% "	55.0	24.8	116	161	179	865	806	15.7	14.6	Yes	Yes		5	5
1298	100% "	48.4	21.8	119	161	203	909	1514	18.7	31.2	Yes	Yes		5	5
1646	100% "	47.5	21.4	122	176	175	742	1041	15.6	21.9	Yes	Yes		5	5
14677	100% "	45.4	20.4	108	202	224	1061	1479	23.3	32.5	No	Yes		6	9
14291	100% "	44.4	20.0	115	196	223	995	1725	22.4	38.8	No	Yes		5.5	7
171	100% "	49.0	22.0	80	228	228	400	1146	8.1	23.3	No	Yes		5	—
1063	100% "	45.0	20.2	106	222	230	656	1409	14.5	31.3	No	Yes		5	5
1265	100% "	44.4	20.0	93	226	239	731	1034	16.4	23.2	No	Yes		5.5	6
9327	100% "	42.4	19.1	109	152	189	1239	1234	29.2	29.1	No	Yes		6	8
9381	100% "	45.4	20.4	106	194	207	855	1305	18.8	28.7	No	Yes		6	8
10063	100% "	50.0	22.5	99	177	210	972	1026	19.4	20.5	No	Yes		6	8
10175	100% "	51.4	23.2	112	164	209	1190	1125	23.1	21.8	No	Yes		5	8.5
10501	100% "	51.4	23.2	96	147	179	1000	1442	19.4	28.0	No	Yes		5.5	8

TABLE II—(*Continued*).

Tests of Papers Made from Sulphite Pulp.
Not Glue-Sized.

Sample No.	Fiber Furnish.	Basis Weight.	Folio Weight.	Strength.	Tear Factor.		Folding Endurance.		Fold Factor.		Sizing.			Formation.	Watermark.
				% Points /Pound.	With.	Across.	With.	Across.	With.	Across.	Glue.	Rosin.	Starch.		
1	100% Sulphite	37.3	16.8	47.0	74	95	4	10	0.1	0.3	Yes		No		
2	100% "	41.8	18.8	80.0	100	86	256	101	6.1	2.4	Yes		Yes		
3	100% "	42.8	19.3	66.0	136	113	71	89	1.6	2.0	Yes		Yes		
4	100% "	44.3	19.9	73.0	70	102	36	94	0.8	2.1	Yes		Yes		
5	100% "	43.8	19.7	62.0	126	146	42	67	0.9	1.5	Yes		No		
6	100% "	37.3	16.8	58.0	93	97	32	32	0.8	0.8	Yes		Yes		
7	100% "	44.9	20.2	52.0	78	80	34	39	0.7	0.8	Yes		No		
8	100% "	43.8	19.7	81.0	115	121	117	368	2.6	8.4	Yes		Yes		

Note: The several units that are employed in Table II are discussed in Industrial & Engineering Chemistry for March, 1931, Geo. A. Richter, p. 266. The figures are all relative and are expressed in terms of the paper industry. "Tear factor" and "fold factor" are given to make suitable corrections for differences in the weight per area of the different samples.

the samples in terms of texture or appearance. In general, papers that have a coarser formation, although manufactured from the same base stock, have higher tear values than do the papers of less mottled appearance. Regardless of this difficulty in comparing the various commercial papers, it is evident from Table II that the papers that are made from good rag fiber and from purified wood fiber are quite superior to those which contain sulphite pulp. The difference is most striking in respect to folding endurance and tearing resistance. A visual examination of the various papers also differentiates the high alpha-cellulose stocks by their distinctive texture and by the rag-like character. The brightness of color and cleanliness of sheet in the case of the best rag fibers and the purified wood fibers are also very noticeable.

RELATIVE PERMANENCE OF FIBER.

There has been much controversy regarding the value of an accelerated aging test in determining the relative permanence of various types of fiber and paper. It is obvious that a one-hundred-year natural aging test would serve no immediate purpose. It is also quite probable that the present-day knowledge of the stability of ancient rag-fiber paper will not necessarily apply to modern paper products, inasmuch as papermaking practice has undergone radical changes during the past forty years. The use of bleach, alum, and rosin has introduced new factors which did not complicate the problem one hundred years ago. Unless a suitable rapid test can be perfected, it would seem that we cannot be assured that any of the papers of present-day manufacture will endure for five to ten decades.

In a rapid test that is designed to indicate the expected life of a paper, it is important that the procedure involve an exposure of the samples to conditions which will intensify the action of those reagents which ordinarily play a part in the natural decomposition of the paper. There is evidence that the most important of the destructive agents are oxygen, moisture, light, acids, and certain metals such as iron.

Although many records and books are exposed to light quite frequently, there are more cases in which the paper is stored in vaults or on shelves and does not come in direct

contact with the sun's rays. In this dissertation the decomposition of paper by light has, therefore, been considered as of minor importance, the more so because it has been established that even strong sunlight causes much slower decomposition than does warmth in the presence of oxygen or acid gases. Furthermore, the slow decomposition of paper in sunlight yields results which for the most part parallel those that may be obtained with the high-temperature tests in a much shorter time.

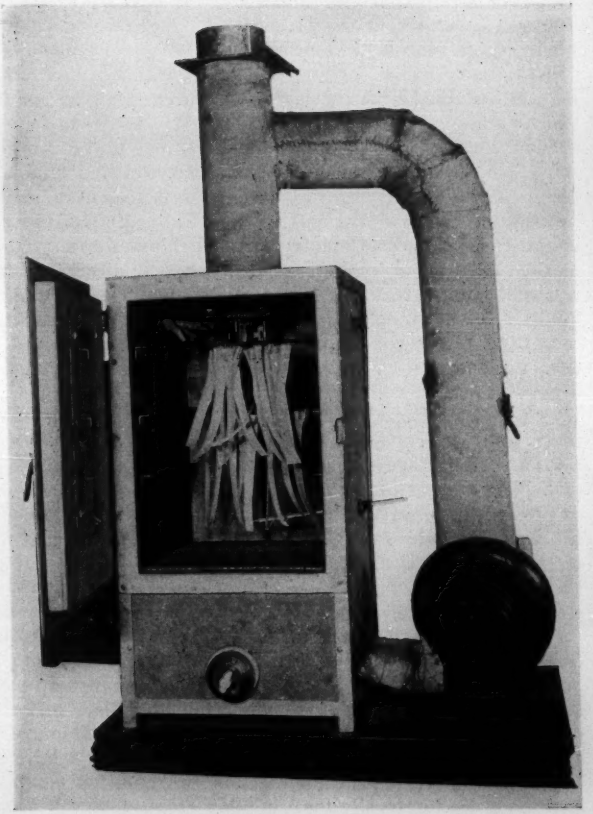
One common test for durability comprises an exposure of the paper to circulating air at somewhat elevated temperatures. A 72-hr. exposure at 100° C. is sufficient to cause measurable physical changes in practically all papers, and in the case of the less stable papers such a test results in a marked decrease in folding strength, which in all cases is the test that is most readily affected. The apparatus which was used for this purpose is shown in Plate XVII. It is possible to substitute a second procedure in which the treatment is carried out at 50° C. and at 200 pounds oxygen pressure for a ten-day period.

This second test was developed to meet possible objections to the 100° C. exposure, which may in some cases cause a partial irreversible dehydration, such as would be unlikely to take place during the course of a natural aging. Inasmuch as practically all of the results from the 50° C. test parallel the findings of the 100° C. exposure, it is concluded that for most cases the 100° C., 72-hr. test is very suitable. In all of the heating tests the relative results were found to be substantially the same, whether the air was dry or moist.

The best criterion of relative permanence appears to be the percentage retention of initial folding endurance when the papers are subjected to the rapid aging and then reconditioned and finally tested. Although the tearing resistance and the pop test suffer some decrease in most cases, the first major change is evidenced by a marked drop in folding endurance and in flexibility. This same behavior is true of papers that have been aged naturally for a number of years.

Most commercial papers change in color when they are aged artificially or naturally. This change in color is, however, not always indicative of decomposition of the fiber

PLATE XVII.



A circulating air oven that is thermostatically controlled for test of permanence of paper.

itself, but may in some cases relate to oxidation of the rosin and the dyestuffs contained in the paper. In the case of so-called "waterleaf" papers, which contain no size or coloring matter, there is a strong relationship between change in color and change in physical strength, but such color changes should not be accepted as the final criterion of deterioration.

Results that are obtained from the rapid aging tests coincide with the usual conception of the durability of the

PLATE XVIII.

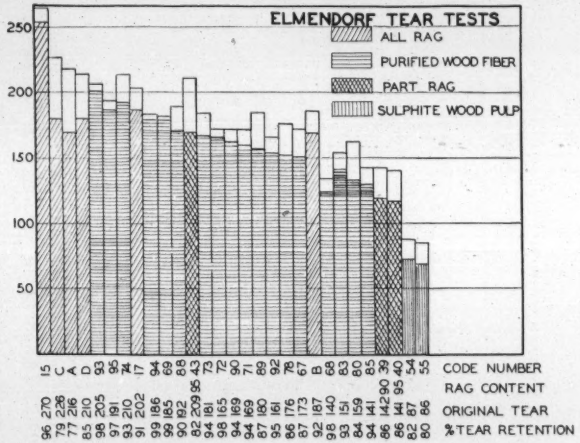


Chart showing original tearing resistance of various grades of commercial papers and loss of tearing strength caused by 72-hr., 100° C. air test. Numerical values of per cent. tear retention are also given.

various grades of paper. In the absence of excess acidity or iron, the various papers group themselves very consistently and confirm the generally accepted idea that a paper which has been produced from a high-quality rag fiber and with reasonable care in the addition of other materials, will endure for much longer periods than will sulphite papers that are prepared with equal care in the paper mill. Papers that contain both new rags and sulphite pulp occupy an intermediate position. Old colored rags that have been drastically

PLATE XIX.

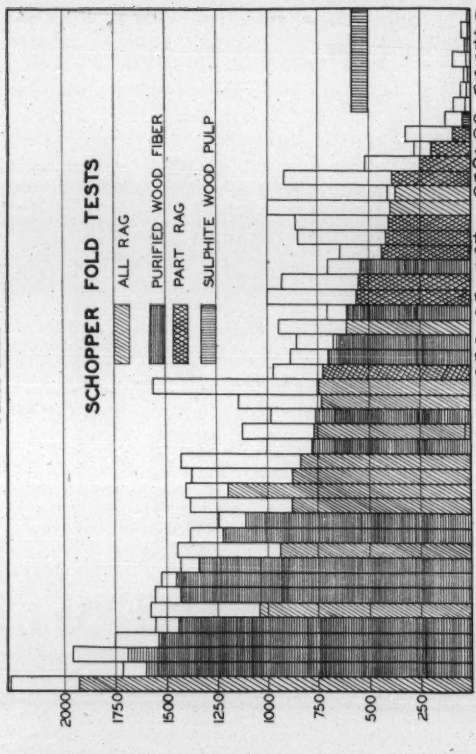


Chart showing original folding endurance of various grades of commercial papers and loss of folding strength caused by 72-hr., 100° C. air test. Numerical values of per cent. fold retention are also given.

treated with chemicals in an attempt to obtain an acceptable color make a paper which is inferior to that made from new rag-fiber, both in terms of permanence and in the initial physical tests.

In all of the accelerated aging tests, the papers which were produced from a well-purified wood fiber were found to be as stable as the best rag papers. This behavior of the purified wood-fiber papers is undoubtedly to be ascribed to their relative freedom from the degraded hemi-cellulose which is abundantly present in the sulphite papers. The tests that

have been made appear to be sufficient in number and sufficiently varied in method of procedure to be quite convincing. The same relative scale of permanence of several kinds of base fiber appears in all of the types of paper which have been examined. This work included an examination of waterleaf papers, papers that had been sized with rosin only, and products which had present both rosin and glue.

PLATE XX.

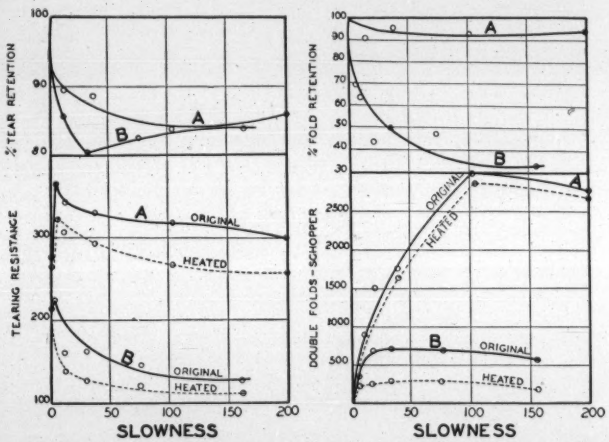


Chart to illustrate changes in tearing resistance and folding endurance of sulphite and purified wood fiber papers produced at different stages of hydration. The several papers were exposed to the 100° C., 72-hr. air test. Resulting physical tests and per cent. retention of tearing resistance and folding endurance are also plotted.

Plates XVIII and XIX exhibit a series of permanence tests that were made with a variety of different types of papers. Both charts represent changes which occurred when the papers were subjected to the 72-hour exposure in circulating air at 100° C. The loss in tearing resistance as well as the sacrifice in folding endurance are given, although it would appear that the retention of the fold or flexibility is of greater significance. The legends are self-explanatory.

Plate XX is of interest in that it illustrates the difference in the fold retention of a purified wood fiber and of a sulphite

pulp when each stock is beaten in the absence of chemicals to increasing degrees of "slowness." The sacrifice in folding endurance of the sulphite pulp becomes more pronounced as the fiber is progressively hydrated to a "slowness" range such as is ordinarily used in papermaking. There is no such marked difference in the retention of tearing resistance of the two types of fiber as hydration is extended. It is to be noted that in all of the above heat tests the sulphite sheets reverted in color very appreciably, whereas the purified wood fiber papers were very little affected.

OXYGEN PRESSURE STABILITY TEST.

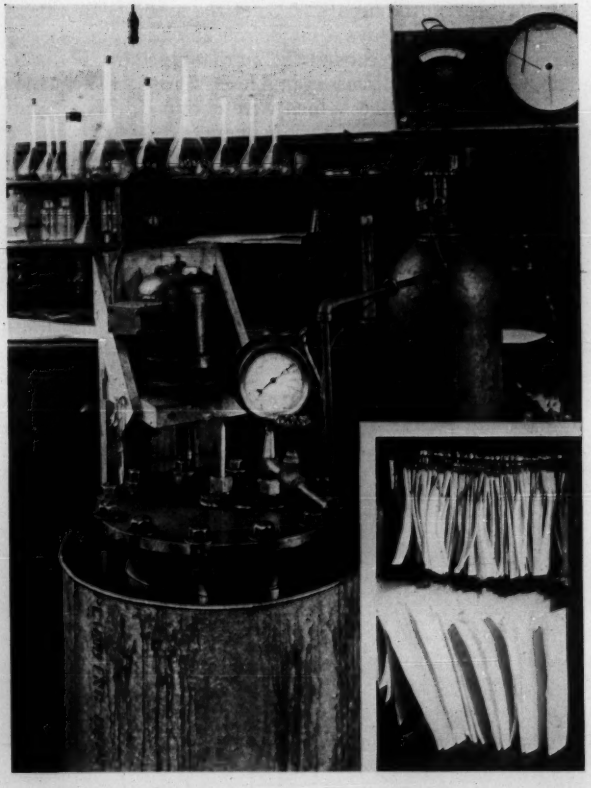
The 72-hr., 100° C. aging test has been criticized as too severe a test and some investigators believe that this high-temperature exposure may cause irreversible changes which do not occur in natural aging of paper. In order to investigate this doubtful effect produced by high-temperature exposure, it was thought best to include other work in which the aging is carried out at lower temperatures and for longer periods of time. To avoid unreasonable length of time, the treatments were made under 200 pounds oxygen pressure and the temperature was maintained at 50° C. and at 60° C. The apparatus that was used for this purpose is shown on Plate XXI. Tests were made in the presence of moisture and also in a dry atmosphere. Under such conditions it was found that a 240-hr. exposure is sufficient to cause major changes to take

TABLE III.

Effect of Dry Oxygen on Physical Tests of Paper.
Treatment with Dry Oxygen at 200 lbs./sq. in. for 240 hours at 60° C.

Type of Paper.	Sizing.	% Retention of Fold.	Color Reversion.
100% Rag Paper (Bond).....	Glue and Rosin	84	Very little
Sulphite Paper No. I.....	Rosin and Starch	65	Appreciable
Sulphite Paper No. II.....	Rosin and Starch	66	Much
Sulphite Paper No. III.....	Rosin and Starch	63	Much
Purified Wood Pulp Paper No. I....	Rosin	85	Very little
Purified Wood Pulp Paper No. II...	Rosin and Glue	83	Very little
Purified Wood Pulp Paper No. III..	Rosin and Glue	91	Very little

PLATE XXI.



Pressure apparatus that was used for testing permanence of paper in presence of oxygen.

place in the less stable papers and to affect even the best papers to some extent. Results which were obtained with the pressure aging procedure appear to parallel those which were secured with the 72-hr., 100° C. test, and indicate that the 72-hr. test can be used to predict the stability of papers. Evidently oxidation plays a major role in the degradation of cellulose composites, although there are unquestionably other factors which will be discussed in a later section of this paper.

Table III gives values which were obtained with pressure tests. In this series the various papers were exposed at 60° C. in an unsaturated atmosphere of oxygen at 200 pounds pressure for 240 hrs.

AGING TESTS AT LOWER TEMPERATURES.

To supplement further the testimony that was established by the 72-hr., 100° C. tests and the 60° C. oxygen pressure tests, various types of commercial papers were aged at a temperature of 37.8° C. (100° F.) in circulating air for a 100-day period. It appears reasonable to assume that stored paper documents frequently experience temperatures of summer heat and that in tropical regions this temperature would usually prevail. The results of this series of experiments are given in Table IV. The data that were obtained again substantiate the findings that were made at higher

TABLE IV.

Results of Fold Retention after 100 Days at 37.8° C.

Type of Paper.	Basis Weight.	Folding Endurance, Average.	Folding Endurance after 100 Days in Air at 37.8° C.	Folding Retention, Per Cent.	pH of Original Paper.
100% Rag Bond No. 1	43.5	1012	804	80	4.6
100% Rag Bond No. 2	44.0	600	533	89	5.2
100% Rag Bond No. 3	47.0	790	658	83	4.6
100% Purified Wood Fiber Bond No. 1.	55.0	631	567	90	5.2
100% Purified Wood Fiber Bond No. 2.	54.2	572	450	78	5.2
100% Purified Wood Fiber Bond No. 3.	55.0	695	593	85	5.2
Sulphite Bond No. 1	44.2	132	81	60	4.4
Sulphite Bond No. 2	43.3	46	24	53	4.8
Sulphite Bond No. 3	45.0	140	106	56	4.2

Note: All tests were made at 45 per cent. relative humidity.

temperatures and lend further confirmation to the theory that the high-grade rag papers and the purified wood fiber papers are distinctly superior to the more impure sulphite papers in respect to resistance to degradation.

DECOMPOSITION OF PAPERS BY ACID REAGENTS.

It is quite certain that oxygen, moisture, and light are not the only reagents which cause paper to deteriorate. The papermaker knows by experience that acids are particularly injurious and he is constantly faced with the problem of producing a suitably sized sheet without the excessive use of alum in the beater. A high acid residue in the paper will accelerate decomposition. There is much evidence that even a very high-quality fiber furnish will prove very disappointing from a standpoint of permanence if excessive alum is used to precipitate the rosin in the sizing operation. This pertains to rag stocks, high alpha wood fibers, and also to the inherently less stable sulphite pulps. For this reason it is unsafe to study the relative permanence of a cellulose by examination of finished papers only. The fiber stability had best be determined on unsized waterleaf papers as were used in a great many of the tests which have been cited above.

It was thought of sufficient interest to demonstrate the susceptibility of all cellulose to acid reagents. This was done by exposing various grades of paper to atmospheres which contained desired percentages of sulphur dioxide. These experiments were carried out at room temperature and in such a way that the sheets to be tested were hung in the respective SO_2-containing air for a ten-day period. In all cases the surrounding air was saturated with water vapor. Check experiments were made under the same conditions except that there was no SO_2 present in the water-laden air. All of the check experiments showed definitely that papers exposed to the saturated air alone did not deteriorate either in respect to tear or fold.

Table V shows very conclusively that the acid atmosphere is very detrimental for all types of paper. The injury is probably caused by a gradual oxidation of sulphur dioxide to sulphuric acid, which is retained by the fiber sheet and which causes hydrolysis to take place even at room tempera-

TABLE V.

Effect of Sulphur Dioxide on Physical Tests of Paper Exposed to Atmospheres Containing Wet SO_2, Room Temperature.

Sample No.	Fiber Furnish.	Grade.	Average Tear Factor.	Average Fold.	Tear Retention Per Cent.	Fold Retention Per Cent.	Original pH.	Final pH.	SO_2 in Atmosphere Per Cent.	Duration of Exposure.
1	100% Rag	—	200	1300	100.0	32.0	5.2	3.6	2	6 days
2	P. W. F. +	—	159	700	100.0	52.0	5.0	3.6	2	6 "
3	Sulphite	Bond	120	175	100.0	36.7	5.8	4.0	2	6 "
4	Sulphite	Bond	100	47	99.0	52.0	5.0	3.4	2	6 "
5	Kraft	—	197	990	100.0	64.0	4.8	3.4	2	6 "
6	Kraft	—	153	1650	100.0	30.6	5.0	3.4	2	6 "
7	Rag	Bond	200	1300	97.0	32.2			5	6 "
8	P. W. F. +	—	159	815	99.6	58.2			5	6 "
9	P. W. F. +	—	180	1020	95.7	31.6			5	6 "
10	Rag	Bond	168	682	97.0	53.0			5	6 "
11	Sulphite	Bond	115	220	100.0	53.0			5	6 "
12	Rag	Bond	210	1250	98.7	70.0			1	10 "
13	P. W. F. +	—	160	680	99.6	78.5			1	10 "
14	P. W. F. +	—	168	920	100.0	75.0			1	10 "
15	Sulphite	Bond	120	218	97.2	73.0			1	10 "

+ P. W. F. = Purified Wood Fiber

ture. In all cases there occurred a decided decrease in pH value of paper, which confirms the conception of a transfer of acidic reagent that is retained by the paper in fixed form. This same phenomenon undoubtedly takes place when paper documents are filed or stored in communities where the atmosphere is contaminated with sulphurous gases, as is the case in most thickly settled districts. Under such conditions no paper will survive except it be loaded with a basic material which will tend to neutralize the condensed acid vapors that are continually brought in contact with the paper.

GENERAL CONCLUSIONS.

It is self-evident that no accelerated aging test will convince an unbeliever that present-day papers will endure the ages as satisfactorily as have some of the ancient rag sheets which are still in existence. Unfortunately we have no way of determining the degree to which these papers have deteriorated from their original physical state. With modern procedures in which color and appearance of paper are stressed far more than was the case two hundred years ago, it would seem probable that many of the papers which are ordinarily considered as permanent will not survive as well as will the papers which are now cited as evidence of the outstanding stability of cotton and linen stocks.

All accelerated aging tests must necessarily be judged on a purely relative basis. It would appear that inasmuch as the commonly recognized perishable papers show a comparatively low stability by the various aging methods which have been proposed, other newer fiber compositions would respond likewise if they were also of the impermanent type. On the other hand, new fiber compositions which simulate the more resistant rag papers in their retention of physical characteristics when exposed to the accelerated aging tests must give positive indication that such new cellulose groups are as stable and permanent as the comparatively high alpha-cellulose rag stocks.

Other work is in progress to supplement our present understanding and to extend the knowledge of those elements which influence the durability of paper and those which cause injury to cellulose material.

New Mechanism Measures Gustiness of the Air. (U. S. Department of Agriculture, Clip Sheet 689.) A gust-recording instrument adapted from the common cup anemometer by C. F. Marvin, Chief of the Weather Bureau, is proving useful in maneuvering the *Akron* in and out of its dock, according to reports from the Akron (Ohio) Airport, where the mechanism was recently installed on top of the weather observation tower near the mooring ground of the giant dirigible.

The modified gust-measuring mechanism consists essentially of an anemometer with three aluminum cups set up on the roof of an observation tower, connected by wires with a recording device in a room below. A magneto attached to the axis of the revolving system of cups generates a current of electricity that varies with the speed at which the cups are spun by the wind. This current, sent on by wires to the room below, moves the pen on a band of paper which passes along fast enough to record in full detail each momentary oscillation of the wind.

R.

THE CONDITIONS FOR IDENTICAL ORBITS IN THE CASE OF TWO CHARGED PARTICLES OF UNEQUAL MASS AND UNEQUAL CHARGE.

BY

W. F. G. SWANN,

Director of the Bartol Research Foundation of The Franklin Institute.

INTRODUCTION.

BARTOL RESEARCH FOUNDATION
Communication No. 60.

The problem which is the subject of this note has a special importance in relation to measurements having to do with isotopes. Its solution is simple, and no claim is made to any priority in respect of it. It is quite possible that several people have worked it out; or, if it has not been worked out, the probable reason has lain in the absence of any immediate necessity for its use at the time. However, the solution does not appear to be given in any obvious place in the literature, and in view of its importance in relation to the work of Dr. K. T. Bainbridge, published in the September issue of the Journal, the present writer has thought it worth while to give the solution as a corollary to that work, and in the hope that it may be of use to others. Dr. Bainbridge himself arrived at the conclusion that for the special case of equal charges with a uniform electric and magnetic field at right angles to each other the condition for identity of orbits was vectorial equality of the momenta of the two particles at some common[1] point of the orbits, for example, the origin; and the present note extends this conclusion to the more general case of an arbitrary static distribution of the electric and magnetic fields, and it also deals with the case where the fields change with the time.

SOLUTION FOR THE STATICAL CASE.

The equation of motion of a particle of mass m_1 and charge e_1 is, for the non-relativistic case,

$$\frac{d^2\mathbf{s}_1}{dt^2} = \frac{e_1}{m_1}\left(\mathbf{E}_1 + \left[\frac{d\mathbf{s}_1}{dt} \cdot \mathbf{H}_1\right] \Big/ C\right), \qquad (1)$$

[1] The equations then, of course, guarantee equality of momenta at all corresponding points.

where $d\mathbf{s}_1$ is an element of the path of the particle expressed vectorially, and $\mathbf{E}_1$ and $\mathbf{H}_1$ are the static electric and magnetic fields.

The corresponding equation of motion of a particle of charge e_2 and mass m_2, in a field of $\mathbf{E}_2$, $\mathbf{H}_2$, is

$$\frac{d^2\mathbf{s}_2}{dt^2} = \frac{e_2}{m_2}\left(\mathbf{E}_2 + \left[\frac{d\mathbf{s}_2}{dt} \cdot \mathbf{H}_2\right] \Big/ C\right). \tag{2}$$

Put

$$\mathbf{E}_2 m_2/e_2 = \mathbf{E}_1 m_1/e_1 \tag{3}$$

and

$$\mathbf{H}_2 = \mathbf{H}_1.$$

Write

$$dt = ad\tau,$$

where a is a constant to be determined. Then (2) becomes

$$\frac{d^2\mathbf{s}_2}{a^2 d\tau^2} = \frac{e_2^2}{m_2^2}\frac{m_1}{e_1}\mathbf{E}_1 + \frac{e_2}{am_2}\left[\frac{d\mathbf{s}_2}{d\tau} \cdot \mathbf{H}_1\right] C. \tag{4}$$

The necessary and sufficient conditions for (4) to be the same equation as (1) are

$$\frac{a^2 e_2^2}{m_2^2} \cdot \frac{m_1}{e_1} = \frac{e_1}{m_1} \quad \text{and} \quad \frac{a^2 e_2}{am_2} = \frac{e_1}{m_1}.$$

Both of these conditions are satisfied if

$$a = m_2 e_1/m_1 e_2.$$

With this value of a, (4) becomes

$$\frac{d^2\mathbf{s}_2}{d\tau^2} = \frac{e_1}{m_1}\left(\mathbf{E}_1 + \left[\frac{d\mathbf{s}_2}{d\tau} \cdot \mathbf{H}\right] \Big/ C\right). \tag{5}$$

The conditions that this equation shall give the same orbits as (1) are that, for the same starting point at $\tau = 0$, the velocities *in the respective systems of time coördinates* shall be equal. In other words,

$$\left(\frac{dx_2}{d\tau}\right)_{\tau=0} = \left(\frac{dx_1}{dt}\right)_{t=0},$$

$$\left(\frac{dy_2}{d\tau}\right)_{\tau=0} = \left(\frac{dy_1}{dt}\right)_{t=0},$$

$$\left(\frac{dz_2}{d\tau}\right)_{\tau=0} = \left(\frac{dz_1}{dt}\right)_{t=0},$$

or, substituting for the $d\tau$ in terms of dt, i.e., putting

$$dt = ad\tau,$$

i.e.,

$$m_1 e_2 dt = m_2 e_1 d\tau,$$

we have, as the conditions,

$$\left.\begin{aligned} m_2 e_1 \frac{dx_2}{dt} &= m_1 e_2 \frac{dx_1}{dt}, \\ m_2 e_1 \frac{dy_2}{dt} &= m_1 e_2 \frac{dy_1}{dt}, \\ m_2 e_1 \frac{dz_2}{dt} &= m_1 e_2 \frac{dz_1}{dt}. \end{aligned}\right\} \tag{6}$$

In other words, the initial momenta of the particles must be proportional to the charges. This condition, combined with the condition that the magnetic field is the same for the two cases and that the electric fields, at each point, are in the ratio of the corresponding values of e/m, in accordance with (3), will insure similarity of orbits for the two cases.

AN ALTERNATIVE DEMONSTRATION.

The result may be reached in a manner which to some will appear more elegant as follows:

The non-relativistic Lagrangian function for a charged particle in a field of scalar potential φ and vector potential $\mathbf{U}$ is

$$L = \tfrac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \frac{e}{c}(\mathbf{U} \cdot \mathbf{u}) - e\varphi,$$

where $\mathbf{u}$ is the velocity of the particle.

The momenta *of the system* are

$$p_x = m\dot{x} + \frac{e}{c}U_x; \quad p_y = m\dot{y} + \frac{e}{c}U_y; \quad p_z = m\dot{z} + \frac{e}{c}U_z. \tag{7}$$

The Hamiltonian function is

$$\mathcal{H} = \sum_{x,y,z}(p_r\dot{q}_r - L)$$

$$= \tfrac{1}{2}m\left\{\left(p_x - \frac{e}{c}U_x\right)^2 + \left(p_y - \frac{e}{c}U_y\right)^2 + \left(p_z - \frac{e}{c}U_z\right)^2\right\} + e\varphi.$$

Now let us consider two particles of charges e_1 and e_2 and masses m_1 and m_2. For the former we have

$$\mathcal{H}_1 = \tfrac{1}{2}m_1\left\{\left(p_x - \frac{e_1}{c}U_{x1}\right)^2 + \left(p_y - \frac{e_1}{c}U_{y1}\right)^2 + \left(p_z - \frac{e_1}{c}U_{z1}\right)^2\right\} + e_1\varphi_1$$

and the Hamiltonian equations with coördinates q_r and momenta p_r, are

$$-\frac{dp_r}{dt} = \frac{\partial\mathcal{H}_1}{\partial q_r}; \qquad \frac{dq_r}{dt} = \frac{\partial\mathcal{H}_1}{\partial p_r}. \tag{8}$$

Let us write

$$\mathbf{P} = \mathbf{p}/e.$$

Then

$$\mathcal{H}_1 = \frac{e_1^2}{2m_1}\left\{\left(P_x - \frac{U_{x1}}{c}\right)^2 + \left(P_y - \frac{U_{y1}}{c}\right)^2 + \left(P_z - \frac{U_{z1}}{c}\right)^2\right\} + e_1\varphi_1 \tag{9}$$

and

$$-\frac{dP_{r1}}{dt} = \frac{1}{e_1}\frac{\partial\mathcal{H}_1}{\partial q_{r1}}; \qquad \frac{dq_{r1}}{dt} = \frac{1}{e_1}\frac{\partial\mathcal{H}_1}{\partial P_{r1}}; \tag{10}$$

put

$$h_1 = \frac{e_1}{2m_1}\left\{\left(P_x - \frac{U_{x1}}{c}\right)^2 + \left(P_y - \frac{U_{y1}}{c}\right)^2 + \left(P_z - \frac{U_{z1}}{c}\right)^2\right\} + \varphi_1. \tag{11}$$

Then

$$-\frac{dP_{r1}}{dt} = \frac{\partial h_1}{\partial q_{r1}}; \qquad \frac{dq_{r1}}{dt} = \frac{\partial h_1}{\partial P_{r1}}. \tag{12}$$

For the particle of mass m_2 and charge e_2

$$h_2 = \frac{e_2}{2m_2}\left\{\left(P_x - \frac{U_{x2}}{c}\right)^2 + \left(P_y - \frac{U_{y2}}{c}\right)^2 + \left(P_z - \frac{U_{z2}}{c}\right)^2\right\} + \varphi_2. \tag{13}$$

Let us put

$$\mathbf{U}_2 = \mathbf{U}_1$$

and

$$\varphi_2 = m_1e_2/m_2e_1\varphi_1 \tag{14}$$

Then

$$h_2 = m_1 e_2 / m_2 e_1 h_1 \tag{15}$$

where each is regarded as a function of the P's and q's, which latter are, of course, x, y, z. Now, the equations of motion of the particle 2 are

$$-\frac{dP_{r2}}{dt} = \frac{\partial h_2}{\partial q_{r2}}; \qquad \frac{dq_{r2}}{dt} = \frac{\partial h_2}{\partial P_{r2}} \tag{16}$$

$$= \frac{m_1 e_2}{m_2 e_1} \frac{\partial h_1}{\partial q_{r2}}; \qquad = \frac{m_1 e_2}{m_2 e_1} \frac{\partial h_1}{\partial P_{r2}}. \tag{17}$$

Now we make the time transformation

$$m_1 e_2 t = m_2 e_1 \tau.$$

Then, from (17), we have, as the equations of motion of the particle 2,

$$-\frac{dP_{r2}}{d\tau} = \frac{\partial h_1}{\partial q_{r2}}; \qquad \frac{dq_{r2}}{d\tau} = \frac{\partial h_1}{\partial P_{r2}}.$$

But, by comparing with (12), we see that these give the same equations of motion as those for the particle 1, so long, of course, as

$$Px_1 = Px_2; \qquad Py_1 = Py_2; \qquad Pz_1 = Pz_2, \tag{18}$$

when, at $t = 0$

$$qx_1 = qx_2; \qquad qy_1 = qy_2; \qquad qz_1 = qz_2.$$

The conditions (18), which we may symbolize as

$$\mathbf{P}_1 = \mathbf{P}_2 \text{ at } t = 0$$

require that

$$p_1/e_1 = p_2/e_2 \text{ at } t = 0. \tag{19}$$

Since $\mathbf{U}_1 = \mathbf{U}_2$, we see from (7) that the condition (19) is the equivalent of

$$\left.\begin{aligned} \frac{m_1}{e_1}\dot{x}_1 &= \frac{m_2}{e_2}\dot{x}_2, \\ \frac{m_1}{e_1}\dot{y}_1 &= \frac{m_2}{e_2}\dot{y}_2, \\ \frac{m_1}{e_1}\dot{z}_1 &= \frac{m_2}{e_2}\dot{z}_2, \end{aligned}\right\} \text{ at } t = 0,$$

which are, of course, the same as conditions (6). These conditions, combined with

$$\mathbf{U}_2 = \mathbf{U}_1 \text{ at corresponding points} \tag{20}$$

and

$$m_1 e_2 \varphi_1 = m_2 e_1 \varphi_2 \text{ at corresponding points} \tag{21}$$

will insure similarity of the orbits for the two particles.

In view of the fact that

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{U}}{\partial t} - \text{grad } \varphi$$

and of the fact that $\mathbf{U}$ is independent of the time, condition (21) is, of course, the equivalent of (3) and (20) is the equivalent of $H_2 = H_1$.

It may be remarked that the simple relationship here developed for the identity of orbits does not hold for the relativistic equation of motion.

$$\left(\mathbf{E} + \frac{[\mathbf{w}\cdot\mathbf{H}]}{c}\right) e = m_0 \frac{d}{dt} \frac{\mathbf{w}}{\sqrt{1 - \frac{w^2}{c^2}}}.$$

Incidentally, if the relativistic Hamiltonian function be approximated to the non-relativistic form, a term mc^2 survives as an addition to the values of $\mathcal{H}$ we have taken. This term, however, contributes nothing to the equations of motion, so we have discarded it.

CASE WHERE THE FIELDS VARY WITH TIME.

Suppose $\mathbf{E}_1$ and $\mathbf{H}_1$ vary with the time. Then, subject, of course, to the correspondence of initial conditions already specified, an equation like (5) will give the same orbit for particle 2 as equation (1) gives for particle 1, provided that, in that modified equation (5), we replace $\mathbf{E}_1$ and $\mathbf{H}_1$ by $\mathbf{E}_1'$ and $\mathbf{H}_1'$ which are the same functions of x, y, z, τ as $\mathbf{E}_1$ and $\mathbf{H}_1$ are respectively of x, y, z, t. In other words, similarity of orbits will be secured if $\mathbf{E}_2$ and $\mathbf{H}_2$ are chosen so that the relation (3) and the relation $\mathbf{H}_2 = \mathbf{H}_1$ hold at corresponding times related by

$$m_1 e_2 t = m_2 e_1 \tau. \tag{22}$$

The condition for similarity of orbits in this respect then involves the distortion of the time scale for $\mathbf{E}_2$ and $\mathbf{H}_2$ in the ratio specified by (22).

STATEMENT OF CONCLUSIONS.

We have thus found that particles of masses m_1 and m_2, and charges e_1 and e_2, starting from the same points, will describe identical orbits provided:

(1) That the initial velocities $\mathbf{w}_1$ and $\mathbf{w}_2$ of the particles are vectorially related at the initial instant by

$$\frac{m_1}{e_1}\mathbf{w}_1 = \frac{m_2}{e_2}\mathbf{w}_2.$$

(2) That, at the same point, but at corresponding times given by

$$m_1 e_2 t_1 = m_2 e_1 t_2$$

the electric fields are related by the condition

$$m_2 e_1 E_2 = m_1 e_2 E_1$$

and the magnetic fields by the condition

$$H_2 = H_1.$$

Granted this, the fields may vary in any manner whatever with x, y, z and t.

Electricity on the Farm. (U. S. Dept. of Agriculture, Clip Sheet No. 691.) New uses for electricity on the farm are being developed constantly as more and more farms are served by power lines, the U. S. Department of Agriculture says.

One of the newest developments is electrical heating of hotbeds. Several agricultural experiment stations have experimented with electric heaters and have found them practical. Electricity is also effective in dehydrating crops. In California it has been found practical in dehydrating for the preservation of nuts.

Farmers are finding that electric power is more satisfactory than high-priced labor in many circumstances. Some of the applications of electricity on the farm are: Feed grinding, milking, threshing, filling silos, cooling milk and cream, driving cotton gins, and pumping water. Farm women use it for a variety of household purposes.

Automatic control of electrical appliances is a desirable feature on the farm. Electricity was first used on farms for pumping water for irrigation, and this still consumes more of this power each year than any other application.

R.

Southern Pennsylvania Topography. (U. S. Geological Survey.) Two areas occupied largely by narrow parallel ridges and valleys in Bedford and Fulton Counties, southern Pennsylvania, are shown on the maps of the Clearville and Hyndman quadrangles, just published by the United States Geological Survey. The varying magnitude of these ridges and valleys depends on the character of the rocks of which they are composed and the manner in which these rocks are bent into great folds that involve beds thousands of feet in thickness. The ridges are formed of rocks resistant to weathering, and the valleys of rocks that are easily soluble and therefore wear down to a very low relief. The hard rocks are sandstone and sandy shale; the more soluble rocks are limestones. The contour lines on the maps, drawn at vertical intervals of 20 feet, give a very clear picture of the corrugated surface, even to an inexperienced map reader. The most prominent ridges are Wills Mountain, which reaches an altitude of more than 2,700 feet, and Tussey mountain, which in several places is higher than 2,200 feet. Town Hill and Rays Hill join to form a feature like the end of a canoe; they are both formed of the upturned edges of a bed of sandstone that is bent into a trough or syncline, which deepens toward the northeast, away from the point where the ridges join. Owing to the roughness of these areas there are no large towns within them, but they are crossed by several State and other highways, including the Lincoln Highway and the Horseshoe Trail.

R.

THE DIFFERENTIAL ANALYZER. A NEW MACHINE FOR SOLVING DIFFERENTIAL EQUATIONS.

BY

V. BUSH, Eng.D.,

Massachusetts Institute of Technology.

1. This paper will describe a new machine for the solution of ordinary differential equations recently placed in service at the Massachusetts Institute of Technology. First, there will be outlined some of the objectives in view, and the general philosophy underlying the development of this type of analyzing device. Various serious mechanical problems have of necessity been met in the effort to produce a machine which is versatile and accurate, and the means by which these have been attacked will be treated. The general method of placing an equation on the machine will be presented and illustrated. Finally, a brief discussion will be given of a method of successive approximations for obtaining solutions of high accuracy, a method for avoiding difficulties due to singularities in coefficients, and a treatment of the problem of obtaining critical values of a parameter in an equation.

2. The handling of the processes of arithmetic by mechanical computation has recently revolutionized business accounting methods. The use in mechanical analysis of more advanced mathematical processes will ultimately be of comparable importance in scientific research and its applications in engineering. This idea is far from original, for Leibnitz envisaged it comprehensively over two hundred years ago. The far-reaching project of utilizing complex mechanical interrelationships as substitutes for intricate processes of reasoning owes its inception to an inventor of the calculus itself.[1] Leibnitz lived in an age when any comprehensive

[1] See "La Logique de Leibnitz," L. Couturat, Chap. IV, p. 115. Leibnitz invented the calculus as one step in a general program of symbolic reasoning. As steps in the program of the mechanization of reasoning he invented a machine

realization of his plan was prevented by lack of the devices and technique by which we are now surrounded. Had he been able to utilize thermionic tubes, photoelectric cells, accurate machine tools, and new alloys, he would certainly have built him many a strange device; for he was a genius and an inventor in the highest sense of the term, and he was fired with the idea of relegating to the machine those parts of the processes of thought which are inherently mechanical and repetitive.

The status of physics and engineering at the present time is peculiarly favorable to a development of this sort. Electrical engineering, for example, having dealt with substantially linear networks throughout the greater part of its history, is now rapidly introducing into these networks elements the non-linearity of which is their salient feature, and is baffled by the mathematics thus presented and requiring solution. Mathematical physicists are continually being hampered by the complexity rather than the profundity of the equations they employ; and here also even a numerical solution or two would often be a relief.

Not any one machine, nor even any one program of development can meet these needs. It was a long hard road from the adding machine of Pascal to the perforated card accounting machines of the present day. There must be much of labor and many struggles before the full ideal of Leibnitz can be consummated.

In the Department of Electrical Engineering of the Massachusetts Institute of Technology the development has followed three lines. First is the process of solving complicated simultaneous algebraic equations as they occur for example in the treatment of modern power networks, by means of alternating-current measurements made on an electrical replica of the power system. The flexible congeries of coils, resistances, and condensers by which this is accomplished is called a network analyzer.[2] Second is the attack

for performing arithmetical processes (see the "Source Book in Mathematics," McGraw-Hill) and an algebraic machine for solving equations. (Various references in Couturat.)

[2] Hazen, H. L., Schurig, O. R., and Gardner, M. F., "The M. I. T. Network Analyzer, Design and Application to Power System Problems," *Trans. A. I. E. E.*, October, 1930. M. I. T. E. E. Research Bulletin No. 66.

on the integral with a variable parameter by an optical method [3] first suggested by Wiener. This gives an approach to the integral equation and to certain processes of statistical analysis. The device itself is called a photoelectric integraph. The third deals with the ordinary differential equation, and provides solutions in the form of plotted curves for specified boundary conditions. The machine treated in this present paper is the latest step [4] along this third line. It is called a differential analyzer.[5]

3. The machine for solving second-order ordinary differential equations which was described in this journal in 1928 has, in spite of its limitations, been successfully used in a large number of problems, some of which have been described in the literature.[6] It has proved very definitely that such a machine, if flexible, rugged, and precise, could be of much

[3] Gould, K. E., "A New Machine for Integrating a Functional Product," *Jour. Math. & Physics*, 17, 305–316, 1929. Gray, T. S., "A Photoelectric Integraph," JOUR. FRANK. INST., July, 1931.

[4] Bush, V., Gage, F. D., and Stewart, H. R., "A Continuous Integraph," JOUR. FRANK. INST., 63–84, 1927. M. I. T. E. E. Research Bulletin No. 56. Bush, V., and Hazen, H. L., "Integraph Solution of Differential Equations," JOUR. FRANK. INST., 575–615, 1927. M. I. T. E. E. Research Bulletin No. 59.

[5] This name was suggested by Prof. W. V. Lyon.

[6] Bush, V., and Gould, K. E., "Temperature Distribution along an Emitting Filament," *Physical Review*, 337–345, February, 1927. Bush, V., "Mechanical Solution of Engineering Problems," *The Tech. Engineering News*, IX, No. 2, 52, March, 1928. Bush, V., "Transmission Line Transients," M. I. T. E. E. Research Bulletin No. 50. Teplow, L., "Stability of Synchronous Motors under Variable Load," *General Electric Review*, 356, June, 1928. Summers, I. H., and McClure, J. B., "Progress in the Study of System Stability," Presented at the Pacific Coast Convention of the A. I. E. E., Santa Monica, Calif., September 3–6, 1929. Gardner, M. F., "The M. I. T. Integraph, Its Operation and Use," Report of the Colloquium on Power-Circuit Analysis, 91–100, June, 1929. Spencer, H. H., "The Solution of a Typical Stability Problem by Means of the Integraph," Report of the Colloquium on Power-Circuit Analysis, 102–112, June, 1929. Lyon, W. V., and Edgerton, H. E., "Transient Torque-Angle Characteristics of Synchronous Machines," Presented at the Mid-Winter Convention of the A. I. E. E., New York, January 31, 1930. Edgerton, H. E., and Zak, F. J., "The Pulling into Step of a Synchronous Induction Motor," *Jour. I. E. E.* (London), 68, No. 405, 1205, September, 1930. Sears, F. W., "A Study of Electron Orbits in the Barkhausen-Kurz Effect," JOUR. FRANK. INST., 209, No. 4, 459, April, 1930. Edgerton, H. E., and Fourmarier, D., "The Pulling into Step of a Salient-Pole Synchronous Motor," Presented at the Mid-Winter Convention of the A. I. E. E., January, 1931.

service in the problems of engineering and physics. It was because of a conviction that such devices are destined to fill an increasing need that the present development was undertaken.

There has been an enormous change in technique since the time when Sir William Thomson first suggested, more than fifty years ago,[7] that the integrators developed by his brother could be connected together and thus forced to produce solutions of differential equations. The idea could then hardly be carried out, for one reason because an integrator, which is simply a variable-speed drive, could not then be built both accurate and capable of carrying sufficient load to move numerous mechanical parts.

The present device incorporates the same basic idea of interconnection of integrating units as did the machine previously described. In detail, however, there is little resemblance to the earlier model. The addition of torque amplifiers has rendered the integrating units capable of carrying a considerable load. A very flexible system of "bus" shafts has been provided by means of which these units can be interconnected or "back coupled" at will. Large size has been preserved in order to obtain accuracy of plotting of the variable coefficients and of the result. Various auxiliary units, such as multipliers, have been provided. The aim has been to produce extreme flexibility, sufficient ruggedness, and reasonable precision. A precision of one part in one thousand under ordinary conditions of use has been arrived at in individual units, with the intention of achieving a somewhat less overall precision except in extraordinary circumstances. The arrangement is still based on the use of integrators because of their inherent average precision, due to the important balancing of momentary errors which occurs in their use. Such effects as are due, for example, to failure of an operator to exactly follow a plotted coefficient are integrated out to an extent which is really surprising, unless one has experienced the balancing of deviations when using a planimeter or steering a ship. A

[7] Thomson, Sir William, "Mechanical Integration of the Linear Differential Equations of the Second Order with Variable Coefficients," *Proceedings of the Royal Society*, **24**, 269, 1876.

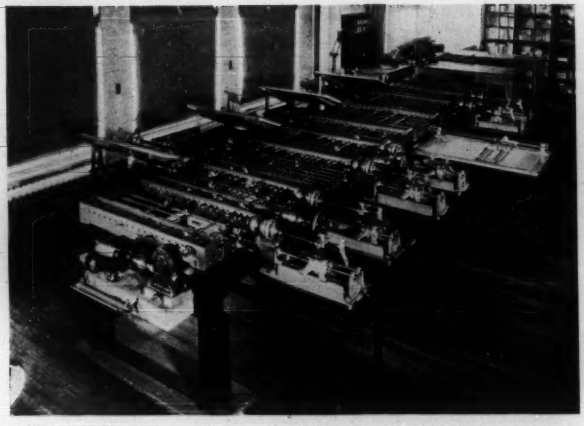
precise differentiator lending itself to automatic operation has never been produced. Electrical integration by watthour meter, very useful in the previous device on account of the possibility of simultaneously multiplying functions, had to be abandoned when precision was extended to the next decimal place in the present machine. All main operations are now mechanical, with electrical devices used only for drives and controls.

The machine as constructed is intended for the solution of ordinary differential equations of any order up to the sixth, and with any amount of complexity within reason. The frequency with which problems are encountered involving two or three simultaneous second-order equations made this range desirable, although it could readily be extended. It is readily possible, when plots have been made and a schematic diagram giving scales and connections prepared, to set up the machine for a given problem in a few hours. The time necessary for solutions varies with the complexity of the problem and the precision desired, and in representative cases is about ten minutes for each solution corresponding to a given set of boundary conditions. Experience is necessary, of course, in order to use the device effectively. This is actually one of the most attractive aspects of the machine; one acquires an entirely new appreciation of the innate nature of a differential equation as that experience is gained.

The attainment of precision in a device of this sort, granted sound mechanical design and accurate construction, is largely a struggle with the problems of backlash and integrator slip, and these will hence receive most attention below. First, however, it will be well to consider briefly the general nature of the units of the machine and the manner in which these various units may be interconnected to solve an equation, assuming that each unit performs exactly as intended.

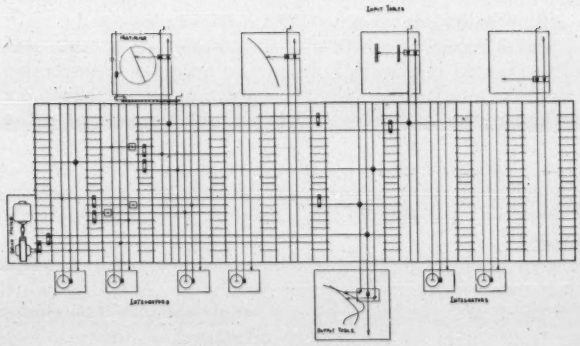
4. A photograph of the machine is shown in Fig. 1, and a schematic diagram of the layout in Fig. 2. It will be noted that there are provided eighteen longitudinal or bus shafts, and that these can be readily uncoupled at many points. Along the sides of the device are ranged the main units: the integrators, input tables, multipliers and output table,

FIG. 1.



The differential analyzer.

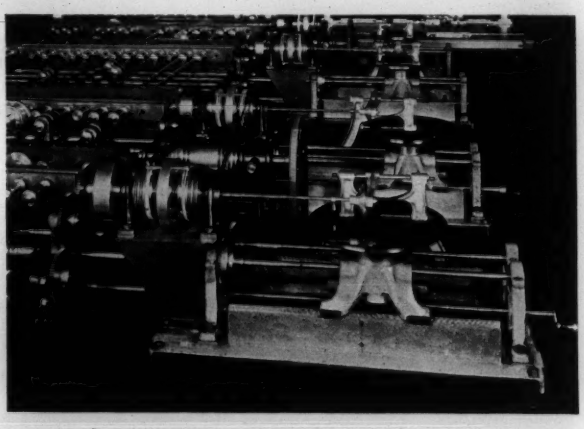
FIG. 2.



Schematic diagram of the analyzer.

each connected to cross shafts. An integrator, Fig. 3, may be considered as a unit with three shafts, the angular movements of which are u, v, and w, so connected that at all instants $u = K\int w dv$. An input table, Fig. 4, has two shafts with revolutions p and q, one of which moves a pointer horizontally in the direction of abscissas and the other vertically in the direction of ordinates across a plot of a function. One shaft is controlled manually to keep the pointer on the plot, thus giving at all times $p = f(q)$. A

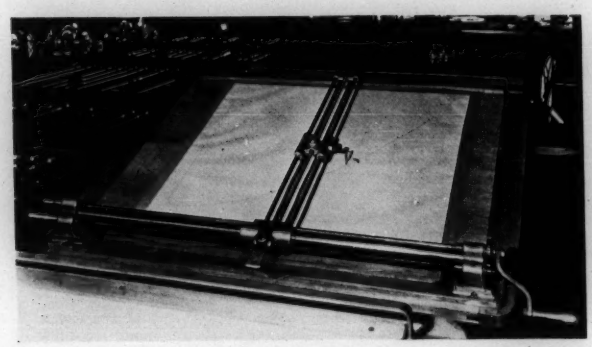
FIG. 3.



An integrator.

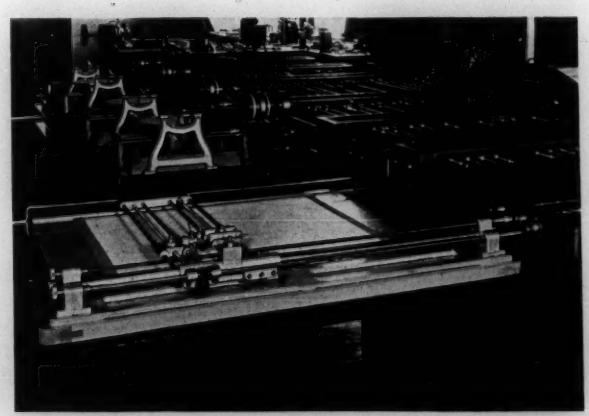
multiplier is obtained by using an attachment on an input table. There are then three shafts, and the revolutions or total angle turned through by one is, at any instant, equal to the product of the revolutions of the other two. An output table, Fig. 5, has three shafts, one of which moves a carriage horizontally, and the other two of which move recording pencils carried by this carriage vertically over the recording paper. Thus two quantities may be simultaneously recorded as a function of any chosen variable. Any cross shaft can be

FIG. 4.



An input table.

FIG. 5.



The output table.

readily connected to any bus shaft by inserting a spiral gear box between them, and one will then drive the other. Right- and left-hand boxes are supplied in order to secure correctly related directions of rotation. One bus shaft, assigned permanently to the independent variable, is driven by a variable-speed motor. There are provided also sets of spur gears which can be connected between adjacent shafts so that one will drive the other with a chosen ratio of speed, and differential gears or "adders" which can be connected to three shafts so that the revolutions of one will be the sum of the revolutions of the other two. These last units are inserted as needed in the body of the table.

It is interesting to contrast the integrators, multipliers, and adders. Each has three shafts. If the angles turned through by these shafts are θ_1, θ_2, θ_3, and their speeds are $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$, then at any instant for the integrator

$$\theta_1 = \int \theta_2 d\theta_3 \quad \text{or} \quad \dot{\theta}_1 = \theta_2 \dot{\theta}_3,$$

for the multiplier

$$\theta_1 = \theta_2 \theta_3 \quad \text{or} \quad \dot{\theta}_1 = \theta_2 \dot{\theta}_3 + \theta_3 \dot{\theta}_2,$$

for the adder

$$\theta_1 = \theta_2 + \theta_3 \quad \text{or} \quad \dot{\theta}_1 = \dot{\theta}_2 + \dot{\theta}_3.$$

It is convenient to have standard symbols for all these various units, and these are shown in Fig. 6.

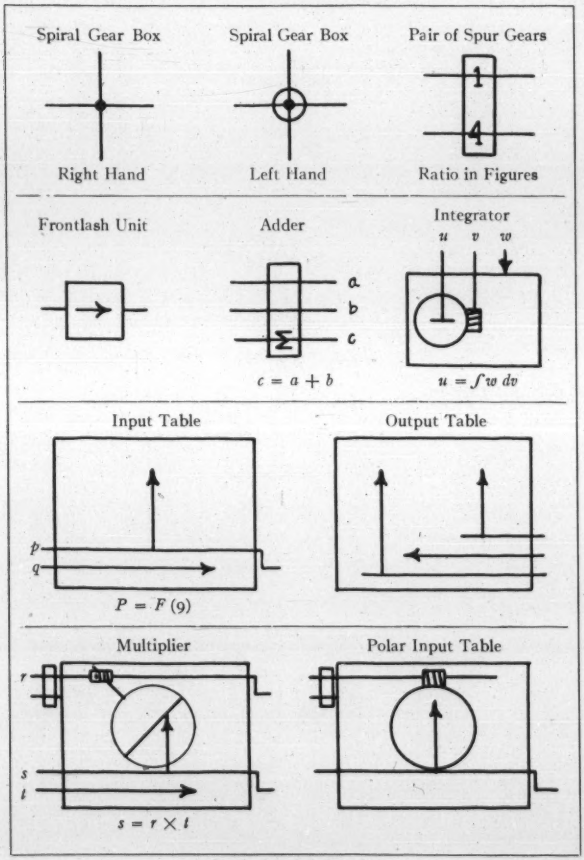
5. Let us now examine somewhat in detail the method of placing a simple equation on the machine. As an example consider the equation

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + g = 0$$

of a body projected vertically in a constant gravitational field, the resistance being assumed proportional to the first power of the speed. Disregarding scales and directions of rotation for the present we may place this on the machine as shown in Fig. 7. The connections will be more apparent if the equation is written in the form:

$$\frac{dx}{dt} = -\int \left(k\frac{dx}{dt} + g \right) dt.$$

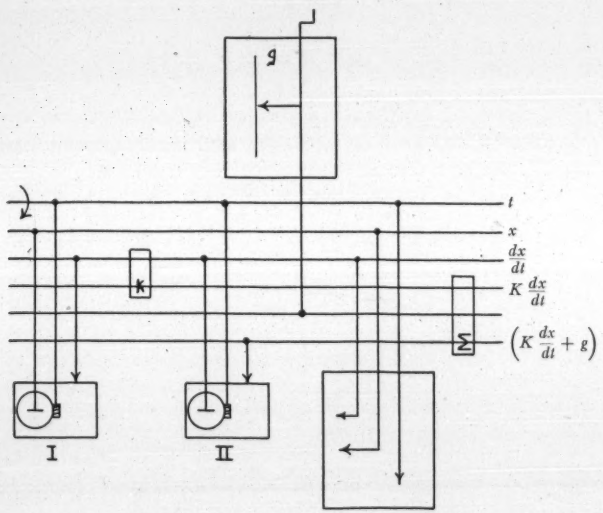
FIG. 6.



Standard symbols for units.

The symbols entered at the end of each bus shaft indicate the equivalent of unit angular rotation of the shaft. We have assumed that k is exactly a certain gear ratio. The procedure, if it is not, will be apparent below. The value of g is, for convenience, placed on an index on an input table where it may be readily adjusted to any desired value. In this problem, of constant coefficients, there are no manual opera-

FIG. 7.



Analyzer connections for simple problem of falling body.

$$\frac{d^2x}{dt^2} + k\frac{dx}{dt} + g = 0.$$

tions. When the time shaft is started, by motor drive, all other shafts are driven. The values of x and dx/dt will be recorded on the output table as functions of the time. The initial values of x and of dx/dt, that is the boundary conditions for the problem, are assigned by specifying the initial displacements of the carriages of the two integrators. For example, suppose we are to start at $x = 0$, $t = 0$, with

$dx/dt = V$. We initially displace integrator I by an amount V, and integrator II by an amount $(kV + g)$ from their mid positions.

Now let us pass to the problem which arises when the gravitational force varies with the distance x, and where the resistance is a complicated but known function of the speed: $f(dx/dt)$. The equation is now

$$\frac{d^2x}{dt^2} + f\left(\frac{dx}{dt}\right) + g(x) = 0$$

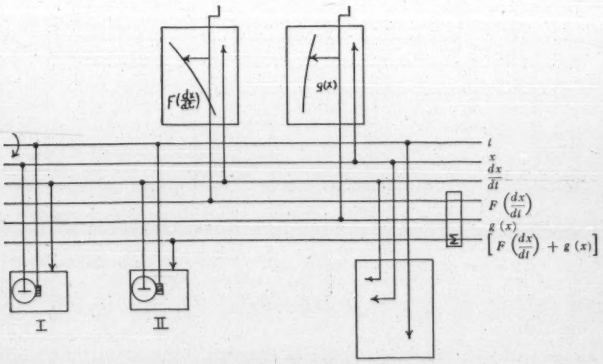
or

$$\frac{dx}{dt} = -\int\left[f\left(\frac{dx}{dt}\right) + g(x)\right]dt.$$

It should be emphasized that, from the standpoint of the machine, the problem is now but little more difficult than before; although formally it may be very troublesome.

The scheme of connections is shown in Fig. 8. Note that

FIG. 8.



Falling-body problem, somewhat generalized.
$\frac{d^2x}{dt^2} + f\left(\frac{dx}{dt}\right) + g(x) = 0.$

we have simply entered the variable coefficients by the use of input tables. An operator is stationed at each of these points, and, as the solution proceeds, he turns a crank keeping an index on a plotted curve. Otherwise nothing is altered.

The machine, as before, causes the distance and speed to be recorded on the output table as functions of the time.

In each of these problems it will be noted that the procedure of placing the equation on the machine is somewhat as follows: A bus shaft is assigned to each significant quantity appearing in the equation. The several relations existing between these are then set up by means of connections to the operating units: a functional relation by connecting the two corresponding shafts to an input table, a sum by placing an adder in position, an integral relationship by an integrator, and so on. When all the relationships which are involved have been thus represented a final connection is made which represents the equality expressed in the equation. In the example above this connection is through an integrating unit, for it is desired to represent the fact that the integral of a certain function is equal to the first derivative of the dependent variable. When this has been done the machine is locked, and the rotation of the independent-variable shaft will drive everything else, thus forcing the machine to move in accordance with the expressed relationship of the equation. The speed of this independent shaft, within mechanical limits, makes no difference. It is, in fact, driven for convenience by a variable-speed motor. This enables everything to be run slowly when an operator is called upon to follow a difficult part of a plot.

The scheme of connecting the machine for a specific problem which has been illustrated is quite general; more so in fact than might at first appear. It has certain features in common with the "plugging" of a desired circuit on a switchboard, and the resulting diagrams have something of an electrical atmosphere about them. Evidently the complexity of the equations which can be handled is limited only by the number of units available.

There will be discussed below more complex situations in connection with matters of discontinuities in coefficients, and successive approximations; and the ways in which precision may be attained in difficult situations will then be apparent. In order that this matter may be clear, however, the design of various individual units will first be treated, with particular attention to precision and reliability of operation.

6. There are six integrating units in the present machine, which form the central feature of the device. Their design constituted a major problem in the development. The task of making a satisfactory integrating unit is that of building a variable-speed drive capable of substantial mechanical power output, accurate in ratio at all speeds and loads, and with this ratio accurately adjustable. Meeting these requirements resulted in a compact rigid mechanical construction, with the backlash in the drives carefully removed, and with a high-ratio torque amplifier added to relieve the friction drive from the necessity of supplying more than a minute torque.

A complete integrating unit is shown in Fig. 3. A massive carriage is moved horizontally on ways by means of an accurate lead screw. This carries a disc in a horizontal plane which can be revolved, independently of the carriage position, by means of splined shafts. Resting on this disc, and pivoted in accurate bearings, is a wheel or roller with its axis parallel to the ways and lying in a vertical plane through the center of the disc. Disc and roller are of hardened steel, ground and lapped. The edge of the roller is given a radius of about 0.002 inch. The outer bearing of the roller shaft is carried by a hinged carriage from the base. This bearing is jewelled and is located directly over the point of contact. End play in roller-shaft and carriage pivots is removable by fine screw adjustment. There are also provided adjustments by which the roller shaft can be brought accurately to its correct position.

Backlash in the lead-screw drive is almost completely removed by using two nuts on the screw, with spring-backed wedges between them forcing the nuts apart, the wedge supports being rigidly fastened to the carriage. By proper choice of the wedge angle this scheme gives a positive drive in either direction, while allowing the slight irregularities of the screw to be taken up without binding. This device, called a "lashlock," was developed by Mr. C. W. Nieman of the Bethlehem Steel Corporation, and, as it has been described elsewhere,[8] will not be discussed in detail here.

The same idea is utilized in the disc drive. On the disc shaft is a spiral gear meshing with two spiral pinions located

[8] *American Machinist*, 66, 921–924, June 2, 1927.

in the carriage and driven by two symmetrically-disposed horizontal splined shafts. This is illustrated diagrammatically in Fig. 9. These shafts are coupled by a pair of spur gears, and at this point is introduced the lashlock. The lashlock-wedge tends to separate the parts against which it presses in such manner as to bring gear teeth into contact all the way around the double drive to the disc. The stud on which the wedge moves is driven by the main disc-drive shaft which projects from the base. Thus backlash in this drive is also eliminated. More care was taken in removing backlash from the integrating units than was perhaps entirely necessary, as will be seen below. It is very convenient,

FIG. 9.

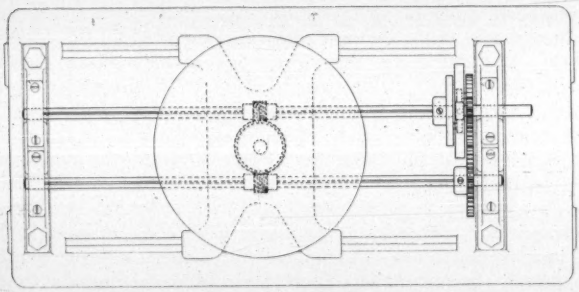


Diagram of integrator drive.

however, to have these integrators precise self-contained units free from substantial errors due to this cause, in order more readily to isolate other possible sources of error.

When the roller is entirely free in its bearings, and when all construction has been made accurately, free from play or backlash, and rigid, the integrator becomes a very precise unit. It takes only an extremely small torque opposing the rotation of the roller entirely to remove this precision. Offhand it might be thought that there would be a threshold value of torque which would cause the roller to slip, and below which precision would be obtained. Actually, however, there is an error caused by any load torque however small; and tests show

that the departure from the correct ratio of disc to roller speeds varies continuously with the torque and is nearly proportional thereto in the range of small values. This effect is undoubtedly caused by the existence of a finite area of contact between disc and roller, at only one point of which is there at any instant zero relative velocity between the surfaces in contact. Such a point will shift its position in the area slightly for any small load increment, and hence cause a small change in drive ratio. It is interesting to note that the error thus caused was found to be substantially the same whether the surfaces were washed free from grease and dried, or thoroughly lubricated with a light oil.

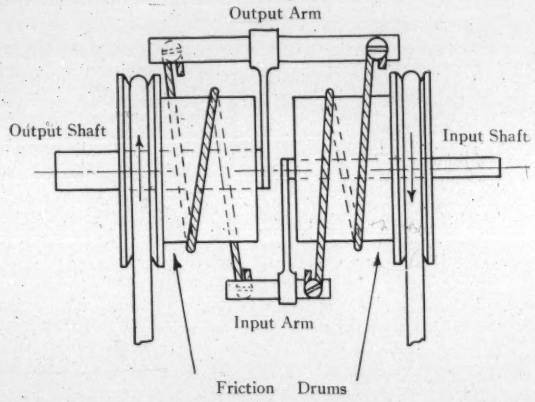
In the previous machine this fundamentally important difficulty was met by the use of servo-mechanisms, the integrating units being then called upon only to move very light contacts, while a motor controlled thereby did the actual work. Unfortunately this almost necessarily involves an oscillatory output drive which is very inconvenient and hardly compatible with extreme precision when complicated interconnection is involved. Thanks to the much-appreciated assistance of Mr. Nieman a much better solution was available for the present machine in the form of the torque amplifier which has been recently developed by the Bethlehem Steel Corporation.[9] This is, in brief, a device having an input and an output shaft, and so arranged that, when the input shaft is turned, the output shaft will turn an equal amount, but with a greatly increased torque, so that a small torque applied to the input shaft is magnified and applied to a load. The energy is supplied by an independent and constantly-running motor which drives a pair of drums in opposite directions. The central idea involved is the use of friction bands wrapped around the rotating drums. A small force applied at one end of such a band produces a much amplified force at the other end, the ratio of these forces varying exponentially with the angle of wrap. The two drums

[9] *American Machinist*, **66**, 895–7, May 26, 1927. *Automotive Ind.*, **56**, 896–7, June 11, 1927. *Chem. & Met. Eng.*, **34**, 443–4, July, 1927. *Elec. Ry. Jour.*, **70**, 18, July 2, 1927. *Mech. Eng.*, **49**, 1017–18, September, 1927. *Power*, **66**, 413, Sept. 13, 1927. *Railway Age*, **83**, 1310–11, Sec. 2, Dec. 24, 1927. *Power Plant Eng.*, **33**, 772–3, July 1, 1929.

provide for rotation in both directions. Fig. 10 shows the basic idea somewhat diagrammatically.

A study showed that about a pound-foot of output torque from integrators would probably be ample, and that, in order that errors due to load might be negligible within the limits of precision aimed at, the torque supplied by the roller could not well be more than a ten-thousandth part of this. It was therefore necessary to develop the torque amplifier along the

FIG. 10.

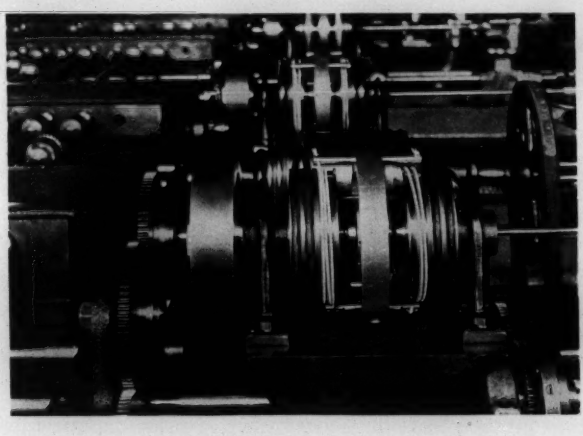


Simplified diagram of torque amplifier.

lines of very high torque-ratio and very low input-torque. A two-stage device was found necessary in order that the bands on the first stage might be exceedingly flexible and thus properly controlled by small forces, while the bands of the second stage might be sufficiently rugged to carry the load. The output of the first stage simply moves a pivoted lever which in turn supplies the input to the second stage. The final model is shown in Fig. 11. The rotating drums are belt-driven in opposite directions, and are stepped to furnish two small-diameter friction surfaces for the first-stage bands, and larger ones for the second-stage bands. There are two concentric shafts. The internal shaft is the roller shaft, held

in a light bearing mounted in the outer shaft, and furnished with an arm which projects through a hole in the latter. This arm carries the input ends of the first-stage bands, which are actually pieces of silk-braided fishline, the output ends of these bands being connected to the pivoted lever mentioned above. The second-stage bands are heavy cord, with their input ends connected to this same lever, and their output ends attached to a ring carried rigidly on the output shaft. One band in each stage is wound clockwise and the

FIG. 11.



A torque amplifier.

other counterclockwise. Every band is given two and one-half turns wrap. There is a very slight initial tension in all the bands, and the effect of a movement of the input shaft is to loosen one pair and tighten the other. The relative angular motion between input and output shafts is only a very few degrees even when carrying full load. This angle has been made very small indeed in certain designs produced by Nieman, but in the design for use with the integrator, where the problem of extremely small input torque is paramount,

it has been found advisable to allow it to be several degrees. The error which might be caused by this small play is avoided by an arrangement described below. The input shaft may, of course, revolve at any speed whatever, and exert complete control over the output, provided the fixed drum speed is not reached.

Now such a torque amplifier is quite analogous to a two-stage thermionic-tube amplifier, and it has many of the properties of the latter, including the possibility of self-oscillation. It was soon found, in fact, that when the amplification of such a low-input unit was raised to around 10,000 it was very prone to go into a condition of violent oscillation ending usually in disaster. This was presumably caused by a small part of the output being fed back in one way or another into the input. This problem caused quite a struggle. It was finally solved by the use of a vibration damper, which is analogous to the introduction of resistance by transformer coupling into the corresponding thermionic-tube circuit. In Fig. 11 it will be noted that there is a relatively massive flywheel on the output shaft. This is loose on the shaft and coupled thereto only through a slight friction introduced by pressure on a felt washer. During uniform rotation the flywheel simply rotates with the shaft. If there are oscillations it slips and introduces a damping force. As in the tube circuit, its presence does not simply limit the amplitude of oscillations; rather it prevents them from starting at all. This scheme, long used for limiting the torsional vibration of machinery, here performs a new function in preventing the initiation of self-oscillations. The performance of the torque amplifiers is now smooth and reliable, and their ratio is ample. It is still necessary to exercise some care in their use, for a spot of dirt on a first-stage friction drum, for example, can cause rough operation. With proper care they give no trouble, and the life of the friction bands is long.

A series of forty test runs was made on a completed integrating unit with various loads of from zero to one pound-foot output torque, at various displacements, and for both directions of disc rotation. The calibration constant was evaluated by averaging all these runs, and the departure for

individual runs then computed. The mean departure was 0.032 per cent., and the maximum departure in any single run 0.12 per cent. This is as good precision as is necessary at this point for all ordinary purposes, although it could now undoubtedly be improved, as the result of experience, if improvement should become advisable.

7. A second major problem in the development has been backlash. A serious matter in any precision device, it becomes especially important in this, because the interconnection of units often renders cumulative the error caused by its presence. The problem is acute when it is desired to produce complete flexibility, for the way in which shafts are geared together is then continually altered.

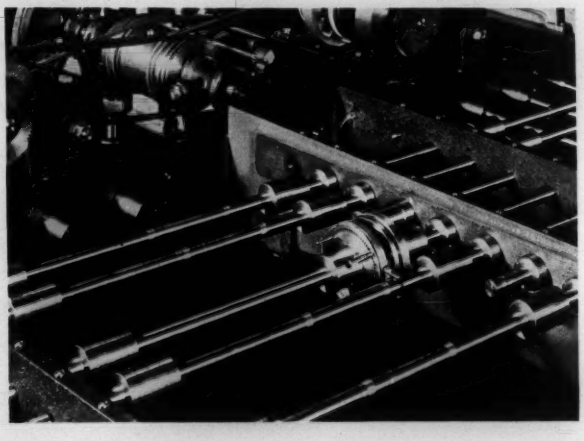
The general scheme of attack has been as follows: Within the drives of integrating units themselves backlash has been substantially eliminated by the use of lashlocks. Elsewhere it has been held down to a small amount by careful construction, although a certain amount of clearance is imperative for the free running of any extensive system of shafts and gearing. In any specific problem the effect of a given backlash angle in a particular drive can then be reduced by specifying scales so that the shafts of that drive will make a large total number of revolutions in the course of a problem. In many situations this is sufficient. Finally, for important drives where the effect of backlash might be especially serious, there has been developed a unit which can be conveniently inserted in any such drive and which reduces the influence of backlash at that point to a second-order effect. This is, in reality, a unit having a negative backlash which can be adjusted to compensate for the positive backlash present in the drive into which it is inserted. It has been aptly termed a "frontlash" unit.

The underlying idea is as follows: The unit, when connected into a line of shafting ordinarily furnishes simply a rigid driving connection from the in-going to the out-going shaft. When the direction of rotation changes, however, it immediately steps the outgoing shaft ahead in the new direction of rotation by an adjustable amount. This it does during part of the first revolution in the new direction. Hence for the balance of the revolutions up to the next

reversal the net backlash in the total drive is brought to zero. This action is repeated at each reversal, so that there is always zero backlash in the shaft, except during the brief periods at each reversal of direction. The effect of backlash itself being small, the residual effect due to this transition period is negligible.

The unit is shown in position in Fig. 12, and disassembled in Fig. 13. A drum is loosely mounted on the in-going shaft,

FIG. 12.

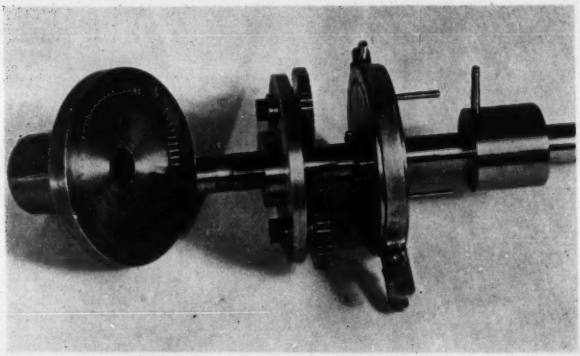


A frontlash unit.

but limited in its rotation with respect thereto by two stops which are readily adjustable. A light friction band, engaging for convenience with an adjacent shaft, rests on the drum and holds it stationary with respect to the base, except when it is caused to rotate in spite of this friction by coming up against one of the stops. A planetary gear train is carried by a mounting fixed on the in-going shaft. One end of this train engages with an internal gear fixed on the out-going shaft. The other end terminates in the drum.

When the drum is up against one of the stops, the whole mechanism rotates as a unit, slipping the friction band. The drive is then direct and none of the gears are rotating with respect to one another, all of course having a common rotation about the main axis. If the direction of rotation of the in-going shaft is now reversed, there is a period before the opposite stop is engaged during which the drum is held stationary by the friction band. During this period the out-going shaft is being driven through the planetary train in such manner that the out-going shaft revolves about ten per

FIG. 13.



Disassembled view of frontlash unit.

cent. faster than the in-going shaft. This continues until the opposite stop engages, when the drive is again one to one. Evidently the actual angular separation of the stops will be about ten times the angular backlash which it is desired to cancel, and hence can readily be set with precision. The friction torque on the drum need be but little more than ten per cent. of the transmitted torque. Hence the load imposed on the drive is not seriously increased by the presence of the frontlash unit.

The usual manner of adjusting a frontlash unit is as follows: It is inserted in a drive, or singly-connected system

of shafts and gearing, in which it is important that backlash should be removed, and usually near the output end of the drive. The drive is marked at each end so that angles may be accurately noted. The input end is then turned through several revolutions in one direction, brought to an accurate mark and the output position noted. This is repeated in the opposite directino. The stops are then adjusted until there is no detectable difference in the position of the output end when a definite position of the input end is thus approached from the two directions of rotation. Gear ratios, if present in the drive, can readily be taken into account. Usually the input end of the drive will be at the roller of an integrating unit. The frontlash unit is then set to cancel the backlash of the torque amplifier as well as that of the balance of the drive. It is of course not always necessary to cancel backlash in this manner. Where it occurs in a drive connected to an input table, for example, it may if necessary be compensated for by adjustment of the plot. Six frontlash units are available, and this appears to be a sufficient number for the present.

A brief study was made of the effect of backlash and its cancellation. For this purpose the machine was set up to solve the simple equation:

$$\frac{d^2z}{dx^2} = -z$$

and the output table was connected to record dz/dx as a function of z, so that the plotted result should be a circle. Since the equation has constant coefficients there were no manual controls involved, and the test was hence unaffected by personal errors. The backlash in the important drive which interconnected the integrators was now measured; and, by what amounted to a perturbation method, its effect on the actual solution calculated. It was found that with this backlash present the machine could be expected to record, in place of a circle, a close spiral with an increment per cycle of about one per cent. in radius. On operating the machine this was indeed found to be the case. A frontlash unit was then inserted and set to cancel this amount of backlash. Again operating the machine it was found to draw accurate

circles, so that if allowed to repeat the record for several cycles the line drawn by a fine pencil point was not appreciably widened. A final test was as follows: The recording pencil was sharpened and allowed to draw one record circle about ten inches in diameter. It was then raised from the paper, but the machine was left running. At the end of an hour the pencil was again dropped on the paper and allowed to draw a second circle. The two circles were accurately concentric, and differed in radius by about one per cent. Even this result could undoubtedly be bettered now, with the better technique in setting frontlash units which has since been developed.

8. There are four input tables, one of which is shown in Fig. 4. They are built to carry plots 18 by 24 inches in size, which allows for considerable precision in plotting. It is possible to increase the precision at this point, as is sometimes necessary, by plotting a curve in several sections, each occupying substantially the whole extent of the table. Successive sections are then brought into play by stopping the machine during the course of the problem and transferring the pointer from one section of the curve to another.

On each table there is a lead screw which is connected with one of the cross shafts and which moves a carriage in the direction of abscissas. This carriage extends perpendicularly across the table in the direction of ordinates, and on it moves a slider carrying an adjustable pointer. A second lead screw located on this carriage moves this slider, and is driven through a pair of spiral gears by another cross shaft of the device. This second drive is also controlled by a crank. Thus, if the first cross shaft is connected to the machine so as to revolve proportionately to a certain variable, and if the crank is turned so that the pointer always registers with a curve placed on the table, the second cross shaft may be connected to pass out into the machine a function represented by this curve and with the variable as argument.

The output table, Fig. 5, is very similar except that there are on the carriage two sliders, each driven by its own cross shaft, and each carrying a recording pencil. The cross shaft which traverses the carriage is ordinarily connected to revolve proportionately to the independent variable, but may be

connected otherwise. Any two chosen quantities may be simultaneously recorded by properly connecting the cross shafts. The usual procedure is to record the dependent variable and its derivative. With simultaneous equations two dependent variables may conveniently be recorded. Sometimes, for example in problems with periodic coefficients, it is expedient to use only one slider and record the dependent variable against its derivative.

For convenience the two pencils are so placed that they record in the same vertical line. This allows both records to have the same abscissas. It requires, however, that provision be made to allow one pencil to pass the other without interference. This is accomplished by mounting one pencil on a spring-controlled plunger attached to the slider, and providing this plunger with a projection which will engage with a similar projection on the opposite slider. Thus, when the pencils come close together, the plunger is caused to recede, and one pencil describes a small arc about the other. The presence of this small arc in the record gives little difficulty.

The lead screws of these tables are interrupted near the ends so that overtravel can not cause a wreck. The connection between lead screws and carriages or sliders is made by a half nut, forced into place by a spring to reduce backlash, and supplied with an arrangement so that it can readily be thrown out of engagement to allow rapid adjustment to a desired position. Final close adjustment is made by moving the pointers in their mountings or by uncoupling and turning cross shafts.

There are two forms of differential gears or adders, and little description of these is necessary. The one shown in Fig. 14 is arranged so that it can be placed in position engaging any three adjacent bus shafts. It is of the planetary type. Gear ratios are so chosen that the revolutions of one shaft are caused to be the sum of the revolutions of the other two without the introduction of multiplying factors.

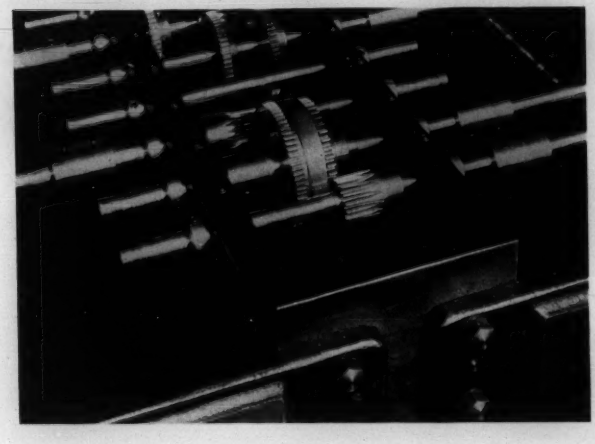
The binary system has been adhered to in nearly all gear ratios, so that the spur gears which may be introduced to connect adjacent shafts are supplied in ratios 1 : 1, 1 : 2, and 1 : 4. By successive steps higher ratios can also be

obtained. Steps as close together as this are necessary for convenience in scales. In order to be consistent with this scheme the factor of the integrating units is made to be exactly 32, so that we then have

$$32u = \int v dw,$$

where u, v, and w are the revolutions of the three cross shafts connected to the unit.

FIG. 14.



A differential gear or adder.

9. An important addition which greatly increases the convenience and flexibility of the machine is a combined polar input table and multiplier. The polar table consists essentially of a large circular plate which can be placed in position on the platen of an input table. This plate can be turned by means of an additional cross shaft by a worm and gear, so that its angular movement is proportional to the revolutions of the cross shaft. The connection as a multiplier will be treated below. There is a handle which is geared to this third cross shaft so that it may, when desired, be turned manually.

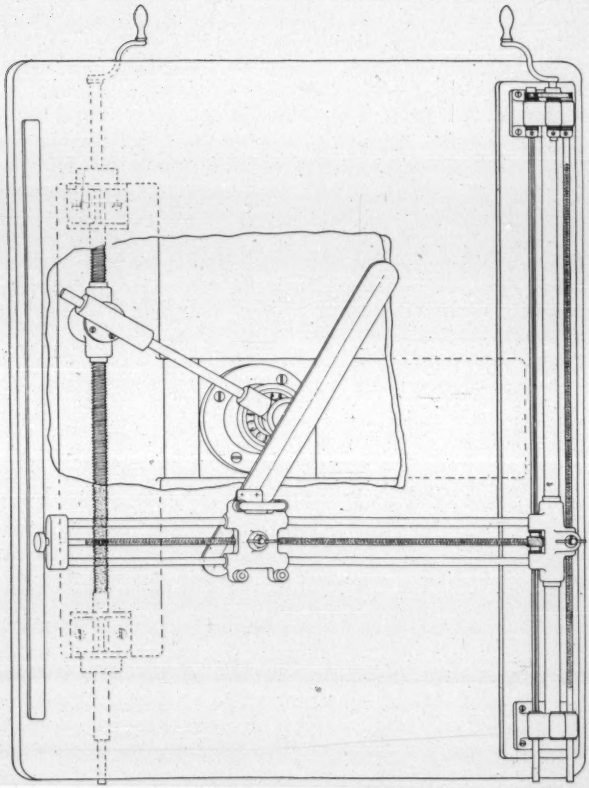
The polar input table has several uses. One use is in connection with periodic coefficients appearing in an equation. Often, in such cases, the operation of the machine will need to be carried on for many cycles of the coefficient; for example, when it is desired to find a stable steady-state solution and to determine a value of a parameter or a set of initial conditions which will produce such a result. The machine can then be left running continuously with the output table set to record a periodic result, such as a plot of dependent variable against its derivative. The parameter is then adjusted until the output is found to exactly repeat cycles, whereupon it may be convenient to alter the connection and record against the independent variable.

For a condition such as this the circular plate is driven from the machine, the input-table carriage is permanently placed accurately in mid-position so that the pointer moves in a vertical line through the center of the plate, and the vertical motion of the pointer is manually controlled so that it is caused to follow a polar plot of the coefficient carried by the plate. The corresponding cross shaft passes this coefficient out into the machine. Gear changes in the plate drive are provided. By the use of these, by the use of plots which repeat several times during a plate revolution, and by proper choice of scales in the balance of the machine, a coefficient of any period may be treated.

A very similar use appears when it is desired to plot a coefficient in much extended manner for purposes of precision. It is plotted in polar coördinates on the plate and winds many times about the center. An inverse procedure is to use a polar input table as an output table, replacing the pointer by a recording pencil, so that a result may be recorded directly in polar coördinates. This again is useful in problems with periodic solutions.

The worm gear drive may be disconnected and the second form of drive mentioned above used in its stead. The circular plate is now replaced by a bar having on its face an accurately scribed line passing through the center. A diagrammatic view of this arrangement is shown in Fig. 15, and a photograph in Fig. 16. In this arrangement there is a lead screw placed parallel to the axis of abscissas and driven

FIG. 15.



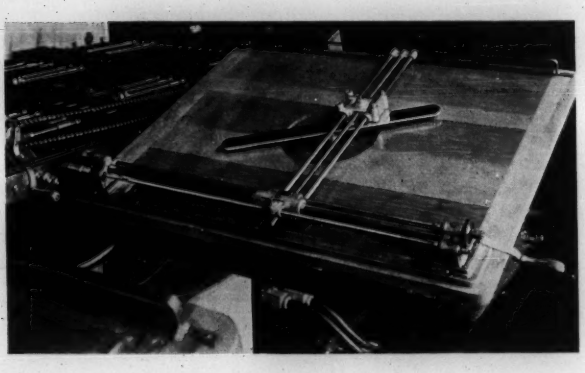
Diagrammatic view of multiplier.

by the cross shaft previously connected to the worm and gear. On this lead screw travels a carriage with a swivelled bearing, and through this bearing passes a rod firmly attached to an extension to the shaft carrying the bar, and located perpendicular to the axis about which the bar revolves. Evidently

with this arrangement the revolutions of the third cross shaft will be proportional to the tangent of the angle turned through by the bar. Call the revolutions of the third cross shaft from the position in which the rod is perpendicular to the lead screw z, and the revolutions of the other cross shafts from the positions in which the pointer lies at the center of the plate x and y. If the manipulation is such that the pointer remains always on the diametrical straight line, we have then

$$y = xz$$

FIG. 16.



A multiplier.

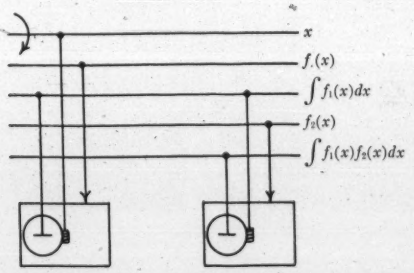
with proper proportionality factors not now considered. There is thus available a multiplier. If x and z are drivén from the machine, and y is controlled manually, a product of two variables is obtained and passed out to the machine. Either x or z can go through zero and take on negative values. If we control x and y by drive from the machine, and control z manually, a quotient can be obtained. Of course in this case x can not go through zero unless y does simultaneously. The choice of scales, and scale changes during a solution when necessary, will maintain the pointer normally at a considerable distance from the center of the plate. This

matter of scales always needs to be considered in connection with the question of precision in the use of an input table or multiplier. The maximum angle with the axis of abscissas through which the bar can turn when the multiplier arrangement is in use is a little over 45 degrees. When this is inconvenient the bar may be unclamped from the rest of the mechanism, rotated through 90 degrees and then clamped again so that the straight line occupies a position perpendicular to the lead screw when $z = 0$. We have then an interchange of variables, so that

$$x = yz.$$

It is always possible to evaluate the integral of the product of two functions by a proper interconnection of two integrating units. The arrangement for accomplishing this is indicated diagrammatically in Fig. 17. In this diagram it is assumed

FIG. 17.



Connections for obtaining the integral of a product.

that $f_1(x)$ and $f_2(x)$ have been introduced into the machine by means of input tables, and integration against x is indicated. The essential point is the connection whereby the output of one integrator drives the disc of the other integrator. Of course any functions appearing in the machine, represented by the revolutions of bus shafts, may be thus multiplied and integrated, instead of these functions of the independent variable used here for illustration. Integration may also be accomplished with any desired variable as the variable of

integration. This scheme for obtaining the integral of a product is very convenient, and can readily be extended to the case where there are more than two factors. Of course, when employed, it cuts down the number of integrators available for other purposes. In some cases it is desirable to multiply functions without immediately integrating the product, as for example when we meet an expression such as

$$\int [f_1(x)f_2(x) + f_3(x)]dx.$$

In such cases the multiplier is necessary, for there is no accurate automatic differentiator available to undo an integration when once it has been performed. The multiplier, moreover, serves a more vital need than this, as will appear in an illustrative example below.

The drive motor is driven by a push-button-controlled Ward-Leonard system. Automatic stops prevent overtravel of integrators and multipliers.

10. The detailed procedure to be followed in placing a problem on the machine belongs in an operating manual rather than in the present paper, but a general discussion of scales and limits is in order, for it is rather essential to a complete understanding of the uses to which the device may be put. There is an art and a technique in its application, just as in the formal treatment of equations, and the technique must be mastered by any one who would become proficient in the art.

The first step is always the preparation of a connection diagram similar to that of Fig. 7 or 8, and some to follow, which shows the essential interrelations only and disregards all scales, limits, and backlash. This is more than a diagram —it is a process of reasoning, and as such it is recommended to those who seek to impart to youth the meaning, as contrasted with the formalism, of the differential equation. An example to illustrate how alternatives arise will be given below.

From this is prepared a second diagram, exactly similar except that undetermined constant factors and gear ratios are introduced. Thus the independent-variable shaft may be labelled Ax, the shaft for the first derivative $B(dy/dx)$, and so on. When these two shafts are plugged to an inte-

grator, the factor of the integrator is immediately introduced, and the shaft to which the integrator output is connected is labelled $(AB/32)y$. Sufficient gear changes are introduced to provide complete flexibility, with the expectation that many of them will later come out 1 : 1 and be eliminated. Thus we would now gear a new bus shaft to the shaft above and label it $n_1(AB/32)y$, where n_1 is some power of 2, thus available by the usual gear changes, and to be determined. On connecting to input or output tables we introduce the factors given by the lead screws and label scales accordingly. When a final connection which expresses the equality is made, there will be introduced a relationship between shafts already labelled, and we shall obtain an equality between undetermined factors which must be satisfied. There can also be then written down a set of inequalities expressing limits. These involve the maximum values of variables, which must needs be estimated, and are introduced in order that nothing can go out of range when carrying out the problem. For example, the maximum number of turns which can be given to the cross shaft which displaces an integrator carriage is 40. We should hence have, following out the above example,

$$B\left(\frac{dy}{dx}\right)_{\max} \leq 40$$

as one of the inequalities. We are now at liberty to assign all our factors arbitrarily, provided we keep within these inequalities. We can usually do so in such a way as to make most of the scales of plots come out to be exact. The assignment of A above determines the length of time for a solution, for the maximum speed at which the independent-variable shaft can be driven is known. Usually A will be picked to be as small as possible, to cut down the solution time, noting, however, that there should be a substantial number of revolutions of every bus shaft of the machine in order to preserve precision. Suppose the output table is connected to this independent-variable shaft through a gear reduction of 1 : 4, so that the label on the axis of abscissas is $Ax/4.20$, where the 20 appears from the lead-screw factor. This means that one inch of abscissa corresponds to $Ax/80$, just as one revolution of the bus shaft corresponds to Ax. If

now we choose A to be 480, the scale of abscissas will be such that 6 inches represents unit value of x. The entire 24-inch range will then allow x to range from zero to four during the course of the problem. Moreover, since 480 revolutions per minute is approximately the maximum speed of drive, a single run through the problem will take about four minutes. The various factors are thus chosen, satisfying the equation between them, and keeping in mind the inequalities. Of course in this process, on a new problem, we do not know at the outset the maximum range of the variables involved, sometimes not even approximately. It is best then to choose factors conservatively so that there is a good probability that the inequalities will be satisfied with plenty to spare, run a first rough solution with the poor precision thus imposed in order merely to determine approximate excursions, and then on the basis of these reassign new factors which will allow good open scales for plots, provide for large numbers of revolutions of bus shafts, utilize integrators and multipliers over a large part of their range, and generally produce precision.

When the factors have been thus chosen a final diagram is prepared showing the actual location of units in the machine. On this are indicated the positions of frontlash units in every critical drive where the presence of backlash might be serious. The direction of rotation of each shaft is indicated, and made correct where necessary by the use of left-hand gear boxes. Plots of coefficients to the proper scales are prepared, and the problem is ready for the machine.

The procedure probably sounds complicated. With practice it can be carried out on problems of moderate complexity in an hour or two, exclusive of the time necessary for plotting. With the final diagram available a few hours more will suffice to connect up the machine, whereupon it is usually employed to obtain a considerable family of solutions before again altering the connections.

11. In this section, and the following, there will be discussed several interesting points which arise in the process of machine solution. Problems will not be treated from the standpoint of the interest in the solutions themselves, for this hardly comes within the scope of the present paper;

but rather it will be the object to illustrate the flexibility of arrangement which is available, and special schemes of avoiding difficulties and securing simplicity and precision. In order to illustrate most readily the Legendre equation,

$$(x^2 - 1)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - \lambda y = 0$$

will be treated, for it brings out excellently several points of interest.

We know, of course, that there are solutions of this equation which are finite in the interval $-1 \leq x \leq +1$ provided λ has a value

$$\lambda = n(n + 1)$$

where n is zero or a positive integer. Let us discuss the manner of handling the equation, however, as though nothing were known concerning it, and we were interested in discovering critical values of λ for which solutions would remain finite over this range, and the corresponding solutions.

The straightforward way of treating the equation proves to be cumbersome, but this method will be treated first for purposes of later comparison. This involves solving for the highest-order derivative and writing

$$\frac{d^2y}{dx^2} = \frac{2x}{x^2 - 1}\frac{dy}{dx} - \frac{\lambda}{x^2 - 1}y$$

or, what is the same thing,

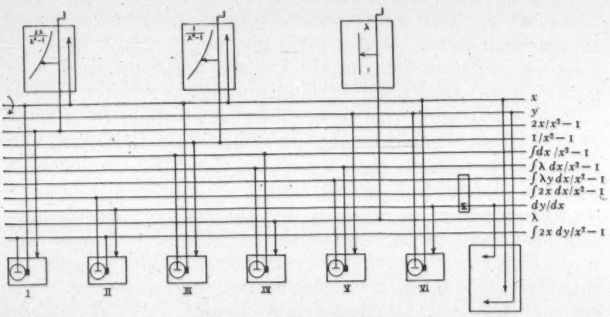
$$\frac{dy}{dx} = \int \left(\frac{dy}{dx}\right)\frac{2x}{x^2 - 1}dx - \int \frac{\lambda y}{x^2 - 1}dx,$$

and using this as a guide in placing the equation on the machine. This results in the diagram of Fig. 18. In preparing this layout no multiplying units were utilized, in order further to illustrate the use of integrators for multiplying purposes. Scale factors are omitted for simplicity, and no attention is paid to signs, limits of travel, or directions of rotation. The arrangement is thought to be clear without discussion except for one matter. The value of λ is placed on an input table, using a scale on this table simply as an index, and multiplication by λ is then obtained by use of an

integrator, the carriage of which thus remains fixed at a certain displacement throughout a single solution. This allows λ to be readily adjustable without altering plotted curves.

Now this arrangement allows us to obtain solutions of the equation for any value of λ and for any given set of initial conditions, provided only that we do not need to examine matters exactly at $x = \pm 1$. We can approach as closely as we please by changing gear ratios and plots every time we

FIG. 18.



Connections for Legendre equation in form:

$$\frac{d^2y}{dx^2} = \frac{2x}{1-x^2}\frac{dy}{dx} - \frac{\lambda}{1-x^2}y.$$

get close enough so that the curves we are using go off scale, but this is likely to be laborious.

It will be well to digress a moment to consider initial conditions. Consider that we are examining solutions in the region from zero to nearly unity, and are starting at $x = 0$. At this point

$$\frac{d^2y}{dx^2} + \lambda y = 0,$$

so that we see there is one set of odd, and one set of even solutions. Let us, to be explicit, consider the second set. These start with zero first derivative, and the solution obtained will be proportional throughout to the initial value

of y chosen, so that we may consider y to start at any convenient value. The initial displacement of integrators I, IV, and V will be zero, that of III and VI will be λ, and that of II will be the initial assigned value of y.

We can now start the x shaft, and run the solution close to $x = 1$. In the immediate neighborhood of this point we have the approximate relation

$$\frac{dy}{dx} = \frac{\lambda}{2x} y.$$

Having carried a solution close to $x = 1$, and recorded dy/dx and y, we now stop and compare the ratio of our final recorded values with $\lambda/2x$. If the ratio checks we know that the solution, if continued, would remain finite up to $x = 1$. Otherwise it would not. If, upon running curves for various values of λ we find two such values which differ by a small amount, one of which makes the final ratio too large, and the other too small, then evidently the critical value of λ which is sought lies between them.

This scheme, of working by the use of approximate solutions valid in the immediate vicinity of points of discontinuity of coefficients, is of general applicability in handling equations having such discontinuities, and is useful when critical values of parameters are sought. An alternative procedure is, however, possible and will now be discussed.

Let the equation be written in the form

$$\frac{d}{dx}\left[(1 - x^2) \frac{dy}{dx} \right] + \lambda y = 0$$

or

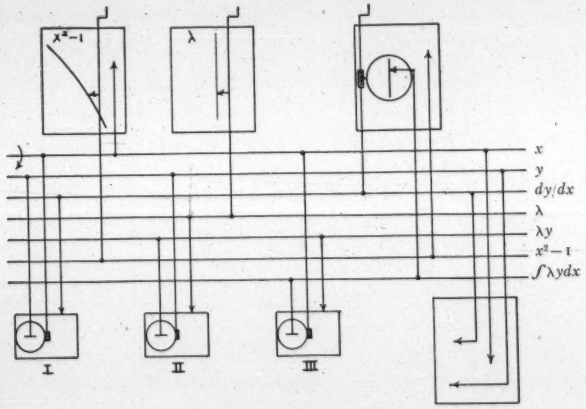
$$(x^2 - 1) \frac{dy}{dx} = \int \lambda y dx.$$

With this as a guide we have an arrangement of connections such as is shown in Fig. 19. A multiplier is now used, and a very considerable simplification has been produced. Moreover we have largely removed the difficulty with coefficients, for a zero point is involved rather than a pole.

If we start a solution at $x = 0$, and examine as before the set of even solutions, the initial displacement of integrator I will be zero, that of integrator II will be λ, and that of

integrator III will be the assigned initial value of λy. As x approaches unity there is now no difficulty provided we have started with a value of λ for which the solution remains finite. As $x = 1$ is approached the pointer on the multiplier will approach the center of the plate, and will finally move in toward the center in a uniform manner so that the operator will have no difficulty in maintaining the line on the plate in register with the pointer even up to the end of the travel. On the other hand if λ is too large or too small the angle will

FIG. 19.



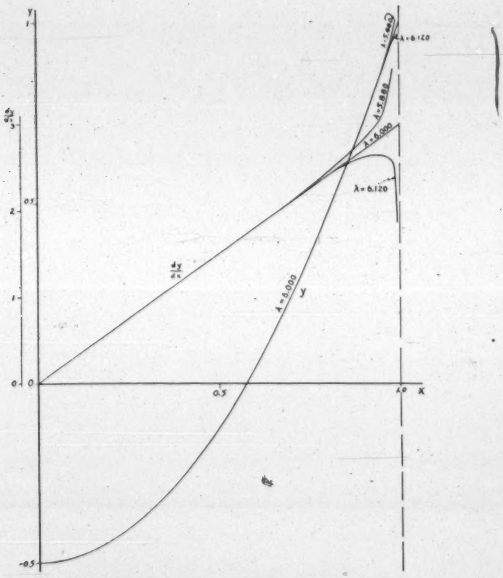
Connections for Legendre equation in form:

$$(x^2 - 1)\frac{dy}{dx} = \int \lambda y dx.$$

change rapidly as the final point is approached. Thus there is no difficulty in identifying critical values of λ within close limits. In Fig. 20 is reproduced a set of solutions obtained in this manner for values of λ in the vicinity of 6, and in Fig. 21 for values near 20. The solutions for the critical value of the parameter are given, and then solutions in which the parameter was altered by 2 per cent. The reproduced curves cannot give, of course, the detail of the original. In the figure no attempt has been made to reproduce the

separate curves except near to $x = 1$, and even in that region the correct details cannot be made apparent at small scale. It was evident, however, from the behavior of the curves for dy/dx that a departure of λ from its critical value to this extent shows up unmistakably, and that critical values can be obtained directly in important cases within rather close

FIG. 20.



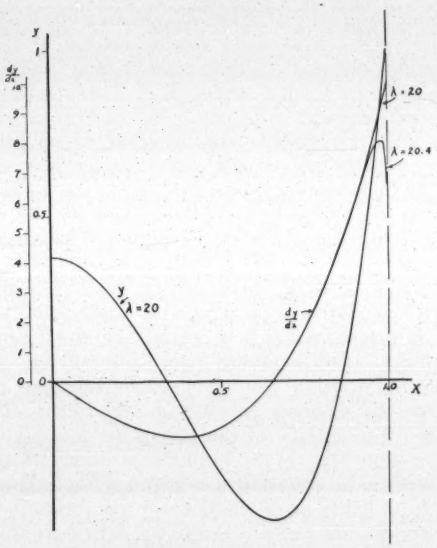
Solutions of Legendre's equation for λ near 6.

limits. This set of solutions was carried out primarily to test the multiplier, and the procedure for avoiding poles of coefficients, and showed that no difficulty is to be expected in approaching a point at which a coefficient goes through zero.

On the other hand we may, if we wish, actually start our solution at the end point. To do this we start at $x = 1$, and run the independent-variable shaft backward. Inte-

grator II starts with a displacement of λ, integrator III with a value λy corresponding to an arbitrarily chosen initial value of y, and integrator I with a displacement corresponding to an initial value of dy/dx equal to $\lambda y/2$ in which y has this same value. The pointer of the multiplier starts at the center of the plate, and the manually-controlled shaft of the multiplier is initially set so as to produce an initial angle for the

FIG. 21.



Solutions of Legendre's equation for λ near 20.

line on the plate corresponding to this same initial $\lambda y/2$. Before starting, in other words, the handle on the multiplier is turned to the position in which the initial displacement of integrator I is brought to the correct computed initial value. Now the instant the machine starts the pointer of the multiplier will move out from the center along the line on the plate, and as soon as it departs from this line visibly correction

is made by the operator, who manually controls the slope of the line in such a way as to maintain it registered with the pointer. There is nothing any more critical in adjustment involved here than at any other part of the solution. In this case, when x reaches zero, we note the recorded value of dy/dx. If it is zero we have so chosen λ as to produce a solution which is finite from $x = -1$ to $x = +1$. If λ deviates from such a value in one direction the recorded value of dy/dx will be positive, and if in the other direction it will be negative. In searching for a critical value of λ we may use interpolation as a guide. Of course the above is written as though we were handling an equation the critical values of the parameter of which were not known, although in this example they are of course well known.

Thus when coefficients appearing in an equation have singularities in the range under investigation it is often best to multiply the equation through in such a way as to produce zeros rather than singularities, whereupon the use of multipliers will ordinarily allow the equation to be treated with little difficulty.

12. As a final topic there will be treated a method of successive approximations which is applicable when an accuracy is necessary which is beyond the direct precision of the machine. It is expected that this will be especially expedient when using the machine to tabulate functions which can be defined by means of linear differential equations. The efficacy of the procedure depends upon the fact that complication of the formal expressions which appear as coefficients in an equation does not then render the machine solution appreciably more laborious or less precise.

The tactics employed are as follows: A first set of solutions is made as accurately as the precision of the machine will allow. Each of these graphical solutions is closely fitted by use of a formal expression which can be easily computed, and with a sufficient number of readily computed derivatives. Then a new equation is set up and solved to give the difference between the functions sought and the formal expressions which were adopted. Finally a numerical combination is made of the exact values obtainable from the formal expressions and the graphically-determined differences from the

second machine solution. The percentage accuracy in evaluating the differences in the second solution will be limited by machine precision in the usual way, but a small error in these small differences will make only a second-order error in the final result.

Thus suppose we have made a first solution which is accurate within a few tenths of a per cent., and that we have fitted formally so that the maximum departure between these solutions and the formal expressions is about one per cent. The values attained by the dependent variable in the second solution will be only about one per cent. of those appearing in the first solution; but by proper choice of scales approximately the full range of plots and units will be utilized in this second equation as well as in the first. We can then expect to evaluate the differences from the second solution to within a few tenths of a per cent. While the situation will be altered in accordance with the nature of the problem it may be then expected that about four-figure accuracy can be obtained from the double solution. Of course the process may be continued another step if necessary.

The second equation is not ordinarily difficult to put on the machine. As an illustration suppose we wished to obtain a very accurate solution of the Legendre equation for a non-integral value of λ over some range within which the solution remains finite. A first solution would be made as described in the preceding section. Suppose this were approximated by a formal and readily computed function $F(x)$, say a polynomial. We then write

$$y_1 = y - F(x)$$

and substitute in the equation. The resulting equation to be solved for differences is

$$(x^2 - 1)\frac{d^2y_1}{dx^2} + 2x\frac{dy_1}{dx} - \lambda y_1 = \phi(x),$$

in which

$$\phi(x) = \lambda F(x) - 2xF'(x) - (x^2 - 1)F''(x).$$

This is but little more difficult to place on the machine than the original equation. All that is necessary is the addition of an input table for introducing $\phi(x)$ at the proper point.

It is not necessary that $F(x)$ be a single formal expression; the first solution may be split up into sections and each represented by a simple formal expression. It is necessary, however, that in each section $F(x)$ have continuous derivatives up to the derivative of an order one less than the order of the equation treated. Otherwise the procedure is not significantly altered from that described above. This method of successive approximations is of course somewhat long, and it will presumably be employed only where the extra precision is vital, but nevertheless it is a very potent method.

13. The machine is not yet completed; in fact it is questionable whether it will ever be complete, for it can always be extended by the addition of units to cover greater order or complexity of equations. It is capable at the present time, however, of handling a wide range of problems of extreme interest.

It has been thoroughly tested for precision. Its speed has been investigated in connection with a problem in which about 100 solutions of a moderately complex second-order equation were evaluated in five days' work. The procedure in locating critical values of a parameter has been somewhat explored. No great experience has as yet been had in work involving successive approximations.

This machine forms part of a departmental program of development. It is a pleasure to recall the effective and enthusiastic support of many of the staff. Prof. H. L. Hazen, who has been associated with me practically throughout the entire development, has consistently contributed in this latest work. Professor M. F. Gardner, and later Mr. S. H. Caldwell have been in direct charge of the laboratory where the work has been in progress, and have greatly helped. I also appreciate the able work of Mr. L. E. Frost, designing draftsman, and Mr. Maurice Forbes, expert machinist.

THE BLACKENING OF PHOTOGRAPHIC PLATES BY POSITIVE IONS OF THE ALKALI METALS.

BY

K. T. BAINBRIDGE,

National Research Fellow.

ABSTRACT.

A short review is included of previous work on the blackening of photographic plates by positive rays and rays of an analogous nature.

The blackening of Eastman x-ray plates, by positive ions, has been measured as a function of the energy of the ions of Li, Na, K, Rb, and Cs. The energy of the ions required to produce a photographic density of $D = 0.3$ with a one minute exposure at a current density of 1.32×10^{-8} amperes per cm.^{2} ranged from 1420 electron-volts in the case of cæsium to 860 electron-volts in the case of Li^7. Approximately 10^5 ions are required at these energies to render developable one silver-halide grain. For a threshold density, $D = 0.04$, under the same conditions the energy ranged from approximately 920 to 460 electron-volts for Cs and Li^7 respectively.

Sensitivity comparisons were made between x-ray plates and process and schumannized process plates. Potassium ions with an energy as low as 137 electron-volts were recorded on Schumann plates, and it is possible that positive ions of one-third this energy can be recorded.

BARTOL RESEARCH FOUNDATION

Communication No. 59.

While there have been many papers on methods of recording positive rays, canal rays, retrograde rays, recoil rays, and mass rays, little quantitative data is available. This paper presents results of a quantitative nature obtained on the registration of positive rays on photographic plates. The apparatus used permitted measurement of the intensity, energy, and mass of the positive rays effective in initiating the phenomena responsible for the blackening of the photographic plates.

Before proceeding with a description of the results, the following rather detailed summary is included of previous results of photographic methods of recording positive rays and rays of a closely analogous nature.

REVIEW OF PREVIOUS WORK.

Koenigsberger and Kutschewski,[1] it seems, were the first to record mass rays on photo-sensitive surfaces. Carbon matt Velox paper was used in their first work and later Velvet Velox was found to be more sensitive. Solarization was produced by prolonged exposures. A perceptible fluorescence was noted on the surface of the photo-paper of about one-tenth the intensity which was produced by the same rays incident on a willemite screen and of about one-third the intensity of the fluorescence from zinc sulphide.

In addition to any mechanical or direct effect of the impacting rays the fluorescent light produced blackening of the photographic emulsion. Thomson [2] demonstrated that the penetration and direct impact of the rays with the silver halide grains played a large part in affecting the plate emulsion. Schumann plates with a minimum of protective gelatine were found to be the most sensitive recording medium but for general use Imperial Sovereign plates were used. Later, Paget process plates were found to be more sensitive than the Sovereign plates. Another fact noted was that hydrogen rays appeared to penetrate furthest into the gelatine. Plates exposed to hydrogen rays required a longer period of development than plates exposed to oxygen rays.[3] Greater density was secured for hydrogen rays than for any other rays of equal intensity and energy.

Retschinsky [4] made use of Schleussner Special Rapid plates. The energy of the canal rays used varied from 5,500 to 38,000 electron volts. With short exposures the plate was blackened after development. Solarization was observed for longer exposures. Also an actual depression in the emulsion was detected at points where solarization had occurred. Retschinsky found that the white solarization spots which were produced at the point of incidence of the most intense part of the canal ray beam could be cleared and made transparent if the plate was put in the fixing bath for a very long time. Six hours were required in some cases to render the white spots trans-

[1] J. Koenigsberger and J. Kutschewski, *Phys. Zeits.*, 11, 666 (1910).
[2] J. J. Thomson, "Rays of Positive Electricity," 2nd Edition, pp. 4–5.
[3] J. J. Thomson, *Phil. Mag.*, 21, 248 (1911).
[4] T. Retschinsky, *Ann. d. Physik*, 47, 525 (1915).

parent. Obviously the rate of diffusion of a solution into the gelatine is decreased markedly as a result of the incident canal rays. The white spots remained white and undeveloped because the developer could not penetrate into the gelatine. Penetration of the fixing bath is secured only after prolonged bathing. A similar effect had been observed by Zehnder [5] in the case of cathode rays. With even longer exposures to intense canal rays at 20,000 volts, black Y-shaped crystalline cracks appeared in the center of the solarization spots. A shrinkage of the gelatine produced Y-shaped cracks which allowed penetration of the developer at these points.

An interesting comparison between papers and plates was made by Jakobson.[6] Jakobson was not acquainted with Retschinsky's work and independently obtained all the results of the latter. Plates were sought which compared in sensitivity with Velox paper. Velox Velvet paper was found to be some ten times more sensitive than the best plates available to Jakobson for positive ray work, in this case the Perutz diapositive plates. These plates have an emulsion similar in composition to the emulsions on Velox and other papers. The Gevaert Ridox paper was superior to Eastman Velox in sensitivity but was capricious in development so that for recording positive rays on paper, Velvet Velox is to be preferred.

Jakobson demonstrated that a large part of the photographic effect of positive rays on plates was due to the light emitted at the surface of the emulsion and that the impact of the positive rays makes the plate lose its sensitivity to light. By exposing plates with the emulsion towards the rays and then with the glass side of the plate receiving the rays it was found that with increased exposure the blackening of the plate increased more rapidly for the fluorescence alone than for the direct effect of the canal rays. The time of development was maintained constant. Velox paper, however, is so insensitive to light compared to any plate that the entire blackening effect must be due to the impact of the canal rays. No blackening was observed as a result of the fluorescence which occurred on the surface of a thin glass plate placed to intercept

[5] L. Zehnder, *Verh. d. D. Phys. Ges.*, **5**, 35 (1903).
[6] M. Jakobson, *Ann. d. Physik*, **73**, 329 (1924).

the rays directly in front of the emulsion of the Velox paper. Jakobson sums up his results on diapositive plates as follows:

1. With exposures of short duration, the canal rays produce a blackening which does not differ materially from the blackening produced by light. With longer exposures this blackening increases to a fixed limit and the maximum blackening produced by canal rays is much less in amount than the blackening which can be produced on the same plate by light. Jakobson concluded that the canal rays only penetrate a short distance into the light sensitive layer.

2. With exposures of still longer duration an effect appeared similar to the solarization observed after long exposures to intense light sources. A white spot appeared in the middle of the black trace where the most intensely concentrated parts of the canal ray beam were incident on the plate.

3. Jakobson rediscovered the star shaped black crosses existent after very prolonged exposures of the plates to canal rays.

Aston [7] made an extensive search for plates which would have greater sensitivity than Sovereign plates for the detection of mass rays, in particular for heavy mass rays. His investigation showed that Paget Half-Tone process plates were quite definitely the most sensitive after a test of a large number of commercially available fast, slow, lantern and process plates. Still greater sensitivity to mass rays was secured by partial elimination of the gelatine from the Paget Plate emulsion, in effect Schumann plates were produced from ordinary commercial plates. A technique was developed for schumannizing ordinary plates by dissolving off the gelatine in dilute sulphuric acid. This method does not differ markedly from the technique developed by Duclaux and Jeantet.[8] As a result of this treatment the plate is rendered greatly more sensitive to mass rays. These plates are especially valuable when dealing with the heavier mass rays.

For the detection of canal rays, Jacobi [9] has utilized the phenomena discovered by Retschinsky, Jakobson, and Zehnder. After exposure to the rays the plate was developed in

[7] F. W. Aston, *Proc. Camb. Phil. Soc.*, **22**, 348 (1925).
[8] J. Duclaux and P. Jeantet, *J. de Physique*, **2**, 154 (1921).
[9] W. Jacobi, *Phys. Zeit.*, **30**, 571, 574 (1929).

bright lamp light. Areas which had received canal rays remained white while the rest of the plate blackened immediately. By the use of a very strong fixing bath the white areas are obtained as transparent patches against a black background. The traces obtained in this way excelled in definition the black traces against a transparent background produced by short exposures followed by development in the usual manner. Traces too weak to appear as transparent areas were detected by photometric examination. Paget Half-Tone and Eisenberger Diapositive plates were used.

Interesting experiments were performed on the direct detection of mercury canal rays at the receiving plate. Traces were visible on a polished aluminum plate cooled by liquid air, although not enough mercury was transferred to detect by a sensitive chemical reaction. Likewise brown traces with a metallic luster were produced on the gelatine surface of an ordinary unexposed photographic plate but no red mercuric iodide appeared, however, when the traces were exposed to iodine vapor. CO_2 traces appeared on the gold plate in addition to the mercury traces so that evidently a change in the surface structure resulted from the impact of the canal rays. The traces were visible on the gold surface without any development such as is necessary to detect the action of low energy electrons on gold surfaces.[10]

Discordance is evident in the results of various authors and very little is known concerning the minimum energy necessary to produce photographic action by positive ions. Knipp[11] determined that 500 electron-volt ions could produce little or no photographic action, and that the first appreciable blackening was initiated by 1500 electron-volt ions.

Walmsley and Makower[12] showed that RaB recoil rays did not give a developable image on ordinary photographic plates. The RaB recoil rays produced by the disintegration of RaA have an energy of 110,000 electron-volts. The failure to obtain a photographic effect in this case obviously resulted from the low intensity of the rays and not from their lack of sufficient energy. This explanation also applies to Jacob-

[10] P. H. Carr, *R. S. I.*, 1, 726 (1930).
[11] C. T. Knipp, *Phil. Mag.*, **22**, 926 (1911).
[12] H. P. Walmsley and W. Makower, *Phil. Mag.*, **29**, 253 (1915).

sen's [13] failure to obtain a photographic effect from α particles having a velocity less than 0.3 of the initial velocity of the α particles from RaC. This velocity of the α particles is equivalent to an energy of 464,000 electron-volts which should be sufficient for photographic action as it is known from the results of Aston, Thomson, and Costa, that He^{++} ions of 10,000 to 40,000 electron-volts energy can produce intense blackening of photographic plates.

Wood and Makower [14] found that even in a good vacuum a fraction of the recoil rays from RaC carried a single positive charge so that some of the recoil rays may be considered as high energy positive rays. Wood and Makower dealt only with the positively charged recoil rays. They detected the direct effect of a RaD recoil beam on Schumann plates. A similar experiment to detect the recoil rays from Po on Schumann plates did not succeed owing to the low intensity of the source.

Smith [15] made a study of retrograde rays by the parabola method of analysis. The minimum energy required for oxygen ions to affect a photographic plate was about 4600 electron-volts. Seed's Gilt Edge No. 27 plates were tried but were abandoned because the faint traces were obscured by the slightest fog which could not be eliminated with such fast plates. The Double Coated Cramer Crown plate was found to be entirely "too sensitive" (to fogging by stray light from the discharge?) so a slow plate giving great contrast was sought. Seed's Yellow Label lantern slide plates were used finally. The retrograde rays (positive rays which have rebounded from the cathode) affected only the surface of the plates as the traces recorded on the plates could be removed from the emulsion surface by gentle rubbing. As found by Koenigsberger and Kutschewski, Jakobson, Thomson, Retschinsky and others, a rapidly moving atom or molecule is just as effective as a positive or negative ion of equal energy for blackening photographic plates.

The recent work of Aston [16] has demonstrated that photo-

[13] J. C. Jacobsen, *Phil. Mag.*, 10, 413 (1930).

[14] A. B. Wood and W. Makower, *Phil. Mag.*, 30, 811 (1915).

[15] O. H. Smith, *Phys. Rev.*, 7, 625 (1916).

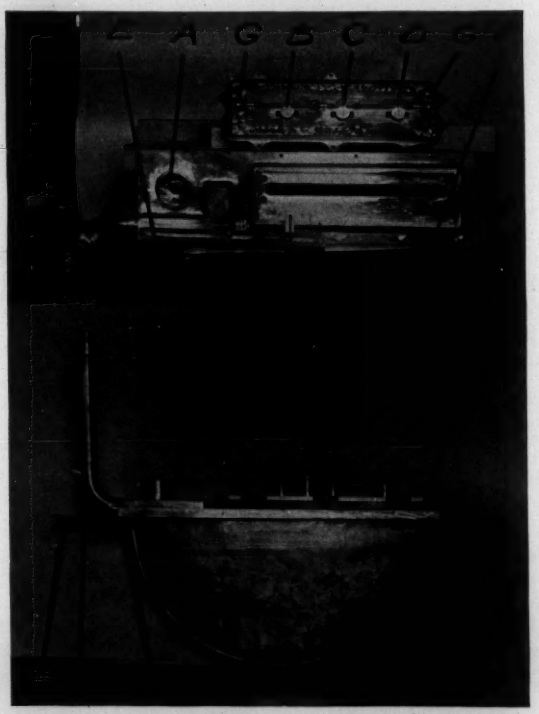
[16] F. W. Aston, *Proc. Roy. Soc.*, 126, 511 (1930); *Nature*, 126, 200, 348, 913 (1930); *Nature*, 127, 233, 591 (1931).

graphic plates may be used successfully for an accurate quantitative determination of the relative abundance of isotopes.

EXPERIMENTAL ARRANGEMENT AND METHOD OF PROCEDURE.

Figure 1 illustrates the tube used for the determination of the blackening of photographic plates by positive ions of the alkali metals. This tube is essentially a Dempster-type [17]

FIG. 1.



Plan and elevation of tube used to determine the blackening effect of positive ions of the alkali metals.

[17] A. J. Dempster, *Phys. Rev.*, 11, 316 (1918).

apparatus for the analysis of positive rays. The tube has been described by the author in a previous communication.[18] The magnet [19] used could provide a uniform magnetic field up to 15,000 gauss with semi-circular pole pieces of 40 cm. diameter separated 2 cm.

Heated natural silicates of the alkali metals provided the positive ions. The ion source, inserted at *A*, was a coated platinum ribbon. The source arrangements used are described in the previous communication to which reference has been made.[20] *B*, *C*, and *D* were collimating slits. Behind these slits were Faraday collectors with leads to a Compton electrometer. The photographic plate used to record the ions measured 25.4 × 2.6 cm. and was placed in the plate holder *F*. A baffle *E* was placed 1 mm. before the plate to collimate the positive ion beams, the separation between the edges was 0.5 cm. By turning the cone *H* the plate holder *F* could be moved parallel to itself by the action of the pinions *G* on the racks visible at each end of *F*.

After the plate had been inserted in the spectrograph, the tube was evacuated. For the purpose of measuring the current density in the beam to be photographed, the positive ion source, the accelerating potential for the ions and the magnetic field were adjusted to bring the positive ion beam to a focus at slit *B*. The width of this slit was 0.033 cm. Only slit *B* was used in this work. The ion current was adjusted so that the peak current density was 1.32×10^{-8} amperes per cm.^{2} when averaged over the slit area (0.033 × 0.5 cm.). The current was measured before and after each exposure. This method of procedure gave an approximate measure of the number of ions incident on the plate emulsion surface, and was accurate enough for the purpose at hand, especially as all exposures taken to compare the effect of ions of different masses, or plates of different types, were made with the constant ion current density given above.

Eastman X-ray, Eastman Process, and schumannized Eastman Process plates were used in this investigation. The proc-

[18] K. T. Bainbridge, J. FRANK. INST., 211, 321 (1931).

[19] This magnet is illustrated in the JOURNAL OF THE FRANKLIN INSTITUTE, 210, 753 (1930).

[20] Reference 18, page 323.

ess plates were schumannized by the method of Duclaux and Jeantet.[8] X-ray plates which were schumannized by this method were unsatisfactory as the emulsion was affected by the temperature and acid to an extent equivalent to exposing the plates to light. Eastman X-ray plates were selected as they have the greatest proportion of silver to gelatine of any plate manufactured by the Eastman Kodak Company. This high silver concentration allowed a reasonable expectation that x-ray plates would be more sensitive than untreated process plates as indeed proved to be the case No comparison was made between Paget process and Eastman process plates. The x-ray plates were developed for five minutes at 18°C. in a glass tube stoppered at both ends, and with a diameter just greater than the width of the photographic plate. The developing solution was agitated violently by shaking and turning the glass tube. The developer was made by dissolving one pair of Eastman No. 2 X-ray developer powders in 350 cc. of distilled water. The process plates were developed in the same developing tube which contained in this case a solution of Eastman D-9 developer.* The schumannized process plates were developed in Eastman D-9 developer diluted with an equal volume of water containing enough potassium bromide to keep the time of development greater than 30 seconds.

The opacity of the traces on the photographic plates due to the positive ions was measured by a Goos and Koch microphotometer. The opacity of a trace is defined as the ratio of the light transmitted through the unexposed section of the plate, to the light transmitted through a trace on the plate. Thus, an opacity of unity represents the opacity of the back-

---

* EASTMAN FORMULA D-9.

Solution A.

Water 52° C.	500 cc.
Sodium bisulphite (E. K. Co.)	22.5 g.
Hydroquinone	22.5 g.
Potassium bromide	22.5 g.
Cold water to make	1.0 liter

Solution B.

Cold water	1.0 liter
Sodium hydroxide	52.5 g.

Use equal parts of *A* and *B* and develop about 3 minutes at 18° C.

ground and fog on the plate. The density is defined as the common logarithm of the opacity.

EXPERIMENTAL RESULTS.

One-minute exposures were taken with singly positively charged ions of Li, Na, K, Rb, and Cs. In all, about 150 separate exposures were made. The background and fog on the x-ray plates were mainly due to scattered light from the filament. When process plates were used, the background was negligible. The typical curves selected for reproduction, of traces on x-ray plates, are directly comparable as these plates were selected for identical background.

The current density was kept constant at 1.32×10^{-8} amperes per cm.^{2} Figure 2 gives the results obtained for Li^7 on x-ray plates. The open and crossed circles represent results obtained on two different plates. Although complete opacity-energy curves were not secured for sodium and potassium ions, sufficient data was secured to determine the lowest energy of the ions which was still effective in blackening the plate. This threshold energy is defined as the energy of the ions which produced an opacity of 1.1 or a density of 0.04 at the current density used and for one-minute exposures. The threshold energies for Eastman x-ray plates expressed in electron-volts were, Cs 920, Rb 760, K 620, Na 500, Li 460. Under the same conditions for an opacity of 2 or a density of 0.3, 1420-volt ions were required for Cs, 1280 for Rb, 1020 for K and 860 for Li.

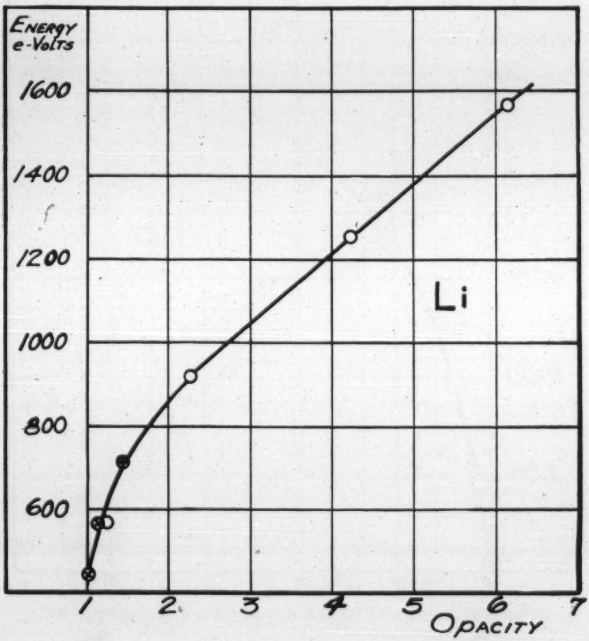
Figure 3 illustrates the opacity-energy results obtained with Rb^+. Figure 4 gives the corresponding data for Cs^+.

The densities varied approximately as the 4.2 power of the energy of the ions, up to a density of 0.3, in the case of Li, Rb, and Cs. At higher densities this exponent decreased. These curves are not reproduced. Figures 2, 3 and 4 show more graphically the dependence of photographic action on the energy of the ions. If the photographic action were due solely to the penetration of the ions in the emulsion, then the density should vary as $(V/M)^{3/2}$, where V and M are the energy and mass of the ions respectively. The density is directly proportional to the mass of silver grains per unit area in the developed photographic plate.[21] If we assume that the

[21] L. A. Jones and E. Huse, *J. O. S. A.* and *R. S. I.*, 11, 330 (1925).

mass of silver grains per unit area which are developed eventually as a result of the penetration of the ions is directly proportional to the range of the ions in the emulsion at low densities, then the density will be proportional to the range of the ions in the emulsion. Since the range is proportional

FIG. 2.

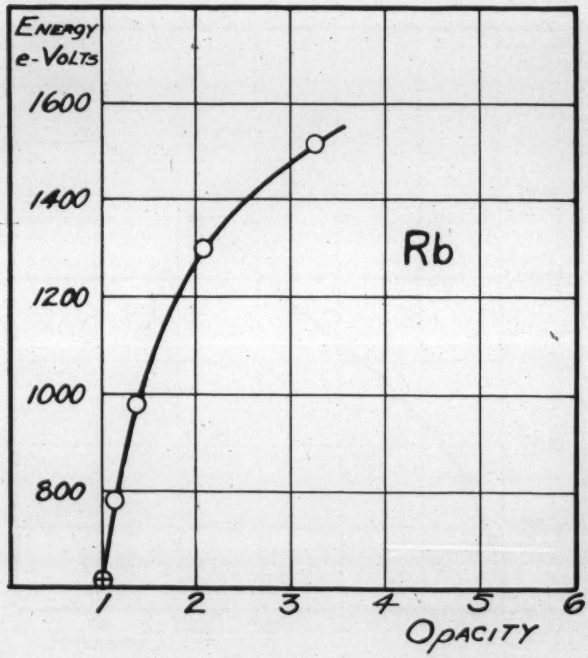


Opacity of Li^7 traces on Eastman x-ray plates as a function of the energy of the positive ions.

to the cube of the velocity or to $(V/M)^{3/2}$, therefore the density should be proportional to $(V/M)^{3/2}$. The densities of the traces actually obtained varied as the 4.2 power of the energy instead of the 1.5 power. Obviously other factors than the results of simple penetration were active in producing the blackening of the plate.

The penetration of ions of such small energy, if it exists at all, is very small compared to the thickness of the emulsion. Light emitted at the surface of the emulsion must play a large part in blackening the photographic plate. Opacities

FIG. 3.



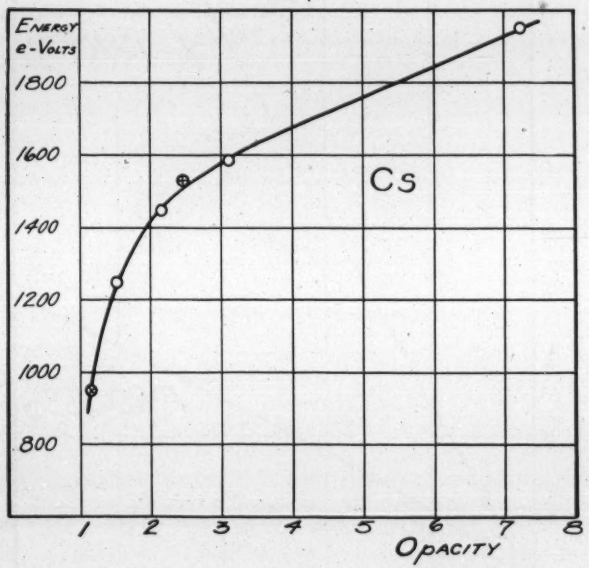
Opacity of Rb traces on Eastman x-ray plates as a function of the energy of the ions.

as high as 29 have been secured with Li^7 and Na at higher current densities and greater energy.

No single law has been found which describes satisfactorily the photographic action of low energy positive ions over the entire range of energies used in the present investigation.

For the case of a density, $D = 0.04$, the factor $V^{3.13}/M$ is a constant for all the alkalies except lithium. For a density of 0.3, $V^{3.8}/M$ is a constant for all the alkali metals except lithium. As has been pointed out, if penetration of the ions in the emulsion is the determining factor $(V/M)^{3/2}$ should be a constant. The effects of charging up of the photographic

FIG. 4.



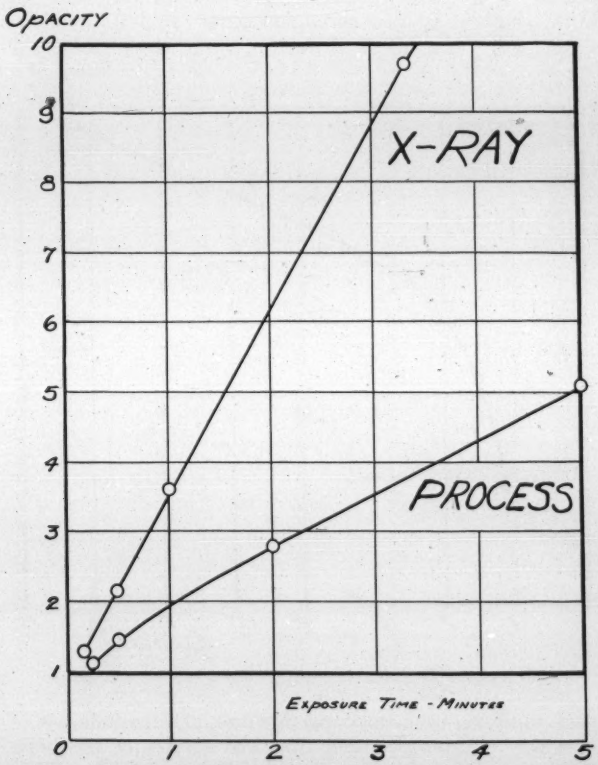
Opacity of Cs traces on Eastman x-ray plates as a function of the energy of the ions.

plate, radiation from the impacting ions, and the influence of secondary electrons emitted from the surface of the brass shield in front of the photographic plate must play a large part in producing the ultimate image on the plate. When dealing with high-energy positive ions, on the other hand, the effects of penetration are readily discernible.[22]

[22] F. W. Aston, *Proc. Roy. Soc.*, **126**, 516 (1930).

Figure 5 gives the results obtained in a comparison of the relative sensitivities of x-ray and process plates to 700-volt

FIG. 5.

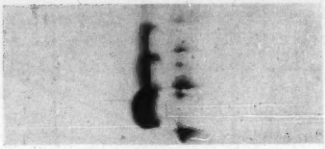


Comparison between Eastman x-ray and process plates. 700 volt potassium ions at a current density of 1.31×10^{-7} amperes per cm.^{2} were utilized.

potassium ions at a current density of 1.31×10^{-7} amperes per cm.^{2} The x-ray plate proved to be approximately three times more sensitive than the process plate. In the particular

case of K ions and this x-ray plate, a linear relation was found between opacity and the product of intensity and time, but this relationship does not generally hold. Curves of opacity against time were taken on x-ray plates for lithium, cæsium and sodium. All of these curves failed to follow a linear relationship at opacities above five. In general the reciprocity law does not hold for the photographic effect of low energy positive ions. Figure 6 is a positive contact print of the 700-volt potassium ion traces on an x-ray plate, which were used for comparing the sensitivity of Eastman x-ray and process plates.

FIG. 6.



700 volt potassium-ion traces on Eastman x-ray plate. Current density 1.31×10^{-7} amperes per cm.^{2} Duration of exposures 10, 30, 60 and 200 seconds.

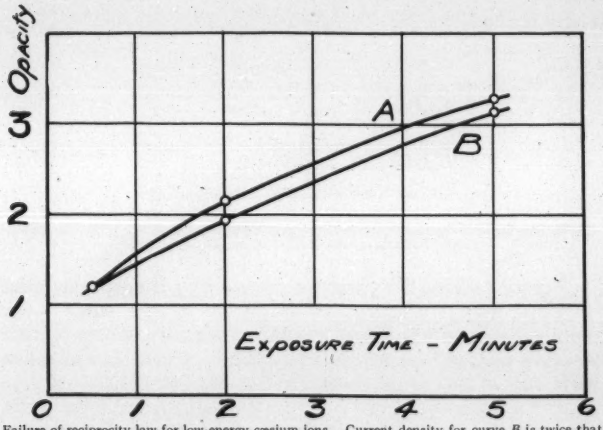
Figure 7 shows the variation of opacity with exposure time obtained with low-energy cæsium ions on the same x-ray plate. Curve *A* was taken at a current density of 0.88×10^{-8} amperes per cm.^{2}, at 880 electron-volts. Curve *B* was taken with ions of 865 volts energy and a current density of 1.76×10^{-8} amperes per cm.^{2} The increased current density did not produce a corresponding denser trace on the plate so that we must conclude that regardless of the current density in the ion beam, only a limited number of ions was effective per unit time.

The results included in Fig. 7 cannot be compared with Fig. 4 as the backgrounds of the two plates were different and therefore the opacities are not simply related. A study of all the exposures taken showed that only those plates could be directly compared which had the same background. No correction for background could be made which allowed the comparison of results from two or more different plates because the blackening due to ions alone was affected by the

background due to scattered light. The effect of exposure of the plate to stray light is somewhat similar to the sensitization of photographic plates to weak sources by fogging the plate slightly preliminary to an exposure.

The great sensitivity of Schumann plates was apparent from some experiments performed with potassium ions. The current density used was 1.32×10^{-9} amperes per cm.^{2} or one-tenth of the usual arbitrarily selected value. X-ray plates required K ions of 620 volts or more to show any effect with

FIG. 7.



Failure of reciprocity law for low-energy cæsium ions. Current density for curve *B* is twice that for curve *A*.

one minute exposures and ten times the above current density, whereas with Schumann plates and ions of 137 volts energy, dense traces were secured even at the low current density used. These plates were not photometered as the background was entirely too uneven for satisfactory work but from a consideration of the shape of the traces it is reasonable to expect that a current density of 1.32×10^{-10} amperes per cm.^{2} would have given traces just as dense, as it appeared that every silver halide grain was developed in the region struck by the ions. With x-ray plates the edges of the traces were sharply

defined and the effect of scattered ions was negligible, whereas with Schumann plates areas near the main traces were dense. Since the current density at those points was certainly less than one-tenth of the current density at the center of the focussed beam of positive ions, the great sensitivity of Schumann plates was demonstrated for ion beams of low energy and small intensity.

Low-energy positive ions are extremely inefficient as blackening agents compared to α and β particles. One α particle can produce between one and ten silver grains upon development of the plate,[23] and only 6 to 8 β particles are required to make a silver halide grain capable of development, whereas the following rough calculation shows that approximately 10^5 positive ions were necessary to produce one developed silver grain. For the purpose of this calculation a density of 1.0 corresponds to a concentration of silver grains[24] of 2×10^8 per cm.^{2} In the present work, data has been given for a density of 0.3 obtained with an ion-current density of 1.32×10^{-8} amperes per cm.^{2} with one-minute exposures. The total ion current per unit area incident on the plate for one minute is equivalent to 5×10^{12} ions per minute per cm.^{2} A density of 0.3 is equivalent to 6×10^7 silver grains per cm.^{2}, so that under the assumption that every ion strikes the plate, $5/6 \times 10^5$, or say 10^5 ions, are required to produce one silver grain in the developed emulsion.

SUMMARY OF RESULTS.

The following results have been demonstrated by the above experiments:

1. Quantitative data have been secured for the response of Eastman x-ray and process plates to positive ions of the alkali metals.

2. Examination of the opacity-energy graphs secured has shown that at the low energies used, in the neighborhood of 1000 electron-volts, penetration of the ions into the emulsion can only play a minor part in the blackening of photographic plates.

---

[23] Hevesy and Paneth, "A Manual of Radioactivity," p. 191. Also ref. 12.

[24] P. Bosch, *Arch. néerland, Sci.*, IIIA, 163 (1925).

3. Eastman x-ray plates have been shown to be some three times more sensitive to the action of positive ions than Eastman process plates.

4. Schumann plates are the most sensitive photographic medium for the recording of positive rays, and it is quite possible that ions of an energy as low as 50 electron-volts can be successfully recorded.

5. For Eastman x-ray plates approximately 10^6 positive ions are necessary to render one silver-halide grain developable at the low energies used in this investigation.

6. In general, the reciprocity law does not hold for positive ions of low energy.

In conclusion, the writer wishes to thank Professor W. F. G. Swann for his interest in this work and for the privilege of working at the Bartol Research Foundation, and to thank the National Research Council for the opportunity to do this research. The writer also takes pleasure in thanking Dr. C. E. K. Mees of the Eastman Kodak Company for his advice, counsel, and aid on several questions dealing with photographic materials.

EVIDENCE FOR THE SECONDARY STRUCTURE OF LITHIUM FLUORIDE BY ATOMIC DIFFRACTION.

BY

THOMAS H. JOHNSON, A.B., Ph.D.,

Assistant Director of the Bartol Research Foundation of the Franklin Institute.

INTRODUCTION.

BARTOL RESEARCH FOUNDATION
Communication No. 56.

The physical properties of matter in the solid state may be classified in two groups, designated by Smekal [1] as structure-sensitive and structure-insensitive properties. The structure-insensitive properties are essentially unchanged by working or by heat treatment, and in respect to these properties polycrystalline substances do not differ from singly crystalline. The structure-sensitive properties, on the other hand are strongly affected by the treatment of the substance, and they vary with the crystalline state. Examples of the structure-insensitive properties are the density, specific heat, compressibility, lattice structure as determined by X-ray analysis, etc. The structure-sensitive properties are exemplified by the elastic limit, the breaking strength, the electric break-down potential, the ionic conductivity of crystals, etc. The structure-insensitive properties have been satisfactorily correlated as characteristics of the primary crystal lattice [2] in which the atoms of the solid arrange themselves, but attempts to explain the structure-sensitive properties on the same basis have led to results which are in error even in their order of magnitude. An example which has been cited by Zwicky [3] of this discrepancy is the breaking strength of rock-salt crystals. A calculation of the breaking strength of the primary lattice gives 20,000 kg.cm^{-2}., whereas the experimental value is only 50 kg.cm^{-2}.,

[1] A. Smekal, *Zeits. f. Physik*, **55**, 289, 1929.

[2] M. Born, "Die Atomtheorie des festen Zustandes," Teubner, 1923. M. Born, and O. F. Bolinov, "Handbuch d. Physik," Bd. 24.

[3] F. Zwicky, *Phys. Zeits.*, **24**, 131, 1923; *Nat. Acad. Sci.*, **15**, 253, 1929; *Helvetica Physica Acta* III, 269, 1930.

and other crystals show similar discrepancies. To account for this low breaking strength Griffiths [4] suggested that even the most perfect crystals occurring in nature are pierced throughout their volume by minute cracks, and Smekal has shown that the same idea can also explain many other structure-sensitive properties.

Zwicky [3] has shown theoretically that a crystal in which there are periodic variations of density is in a lower energy state than a perfectly continuous crystal and he has derived the approximate expression for the period D of the density variations,

$$D \sim d/(1 - d_E/d), \qquad (1)$$

in which d_E is the lattice constant of an ideal plane array of atoms segregated from the rest of the crystal, and d is the lattice constant of the three-dimensional lattice. In the case of the alkali halides Zwicky [3] and Lennard-Jones [5] have shown that d_E is about 6 per cent. less than d and hence $D \sim 17d$. This distance is, of course, outside of the range of the microscope. In the case of metal crystals, however, the contraction $d - d_E$ is considerably less, and in some cases, D is, within the range of microscopic detection. In fact Zwicky, Hasler and Goetz [6] have found three sets of lines intersecting at 60° on a freshly cleaved [111] face of a Bi crystal. The base of the smallest triangle formed by these lines was 1.4×10^{-3} mm. with larger spacings occurring at integral multiples of the smaller unit. This spacing is of the right order of magnitude to agree with D of Zwicky's theory although in the case of Bi the contraction $d - d_E$ is not exactly known. The action of acid in etching this face indicated that the atoms on the edges of the triangles had a lower chemical affinity than those at the centers, suggesting a surface contraction. These experimental results are therefore in accord with the secondary structure as described by Zwicky. Other less direct experimental evidence also exists supporting the idea of the secondary structure of Bi and of other crystals.[7]

[4] A. A. Griffith, Proc. Int. Congr. Appl. Mechanics, Delft 55, 1924.

[5] J. Lennard-Jones and B. M. Dent, *Proc. Roy. Soc.*, **121**, 247, 1929.

[6] A. Goetz, *Proc. Nat. Acad. Sci.*, **16**, 99, 1930.

[7] For a discussion of this evidence see the papers of Zwicky and Goetz already cited.

In the case of the alkali halide crystals, however, there has been up to now, no direct experimental evidence to support the theory of a secondary structure. The spacing to be expected for the variations of density is below the resolving power of the microscope, and the volume within which the density is appreciably lowered should be too small to produce a detectable intensity in the X-ray pattern characteristic of the secondary structure. It seems as if it should be possible to obtain a plane-grating spectrum of soft X-rays if they were reflected within the angle of total reflection from the surface of a crystal upon which the secondary structure has been developed by etching, but this has not been tried. Electron waves, like hard X-rays, are reflected from within the volume of the crystal and hence the detection of the secondary structure, by electron diffraction is not to be expected.

The de Broglie waves of the atom, on the other hand, are reflected strictly from the surface of the crystal and the diffraction patterns obtained by the author [8] when atomic hydrogen was reflected from the cleaved [100] surface of an LiF crystal, contained, in addition to the primary spectrum from the square array of ions on the surface, a secondary spectrum arising from a secondary periodicity in the reflecting power of the crystal surface which may correspond to the secondary structure of Zwicky's theory. The investigation of this secondary spectrum in the case of LiF has now been carried to what appears to be its ultimate conclusion, and it is the purpose of this paper to present the results of this work.

EXPERIMENTAL EVIDENCE.

The experimental arrangements were essentially the same as those which have previously been described, and therefore for a detailed description of the apparatus and of the experimental procedure the reader is referred to earlier papers.[9] A sharply defined beam of atomic hydrogen from a Wood discharge tube was collimated by a series of tubes with suitable pumping systems for eliminating the excess gas. This beam was reflected from the crystal surface, and the diffraction pattern was recorded on a molybdenum oxide surface. The

[8] T. H. Johnson, *Phys. Rev.*, 37, 847, 1931; 37, 87, 1931.

[9] T. H. Johnson, loc. cit.; JOUR. FRANK. INST., 210, 135 (1930).

atoms in the incident beam had the velocities of thermal agitation at room temperature, and hence the beam was not homogeneous in de Broglie wave-lengths, but it contained a distribution in which most of the intensity was included between 0.5 A. and 2.8 A. with the most probable wave-length equal to 1.1 A.

The patterns showed two types of diffraction, the more striking of which were the primary spectra from the square array of ions which constituted the surface of the primary crystal lattice. These spectra have been fully described in the papers to which reference has already been made. In addition there were the secondary spectra of low dispersion produced by a secondary periodicity in the reflecting power of the surface. With the incident beam at an angle of 45° with the surface of the crystal, these spectra appeared as short blurs running out from the specular beam in well-defined directions which were characteristic of the spectra from a grating, the lines of which were intersections of [001] and [010] planes with the [100] surface plane. Thus when the incident beam lay in a [110] plane, the secondary spectra appeared as the arms of a blurred *X*, whereas with the incident beam in a [100] plane, the spectra had the form of an upright cross with the vertical arm in the plane of incidence. The existence of these well defined directions of the secondary spectra was the strongest indication that the phenomenon involved was diffraction and not a reflection from slightly tilted segments of the surface, for there was no reason to suppose that these segments should have been tilted such that their normals lay only in the [001] and [010] planes. The idea that the secondary spectra were produced by a diffraction phenomenon was further strengthened by the behavior of the dispersion of the patterns at grazing incidence. An exposure at about 5° from grazing in the [010] azimuth resulted in a secondary pattern in which the branches in the plane of incidence were considerably more dispersed than the branches at right angles to this plane. Moreover the difference in the dispersions of the vertical and horizontal branches was greater in the grazing incidence pattern than in the 45° pattern. This behavior of the phenomenon is, of course, just what is to be expected of a plane cross grating diffraction and it would be difficult to explain in any other way.

There was some variation in the relative intensities of the primary and secondary spectra, appearing to arise from variations in the heat treatment given to the crystal for the purpose of degassing the surface before and during the exposure. These variations are of significance in connection with the explanation of the secondary pattern. In general it was found that the more vigorous the heating, the more intense was the secondary and the less intense was the primary pattern, as long as the heating was sufficient to degas the surface. A preliminary heating for three minutes at 500° C. followed by a steady heating at 300° C. during the exposure, gave an intense primary pattern, but a much longer preliminary heating at 500° C. followed by the steady heating at 300° C. during the exposure had the effect of weakening the primary and intensifying the secondary spectra. It may be inferred, therefore, that the more or less periodic variations in the reflecting power of the crystal surface which were responsible for the secondary spectra were produced by the etching effect of the heat treatment.

In view of the fact that Zwicky, Hasler and Goetz had found a uniformly spaced set of cracks on the face of a Bi crystal, it seemed possible that the variations of reflecting power on the face of the LiF crystal, which had produced the observed secondary spectra, might be sufficiently uniform to be capable of producing a well resolved spectrum of the de Broglie wave distribution if the resolving power as limited by dispersion and slit widths could be increased sufficiently. If this were the case it would of course be possible to obtain the lattice constant of the secondary structure by measuring the dispersion of the most probable wave-length. With this end in view the resolving power of the slit system was increased and higher dispersion was sought at angles of incidence still closer to grazing.

In the earlier work a beam of circular cross section had been used, but with the higher resolving power it was necessary to improve the intensity by the use of a ribbon-shaped beam. The source slit was the same as had been used for the production of a circular beam, but the collimating system was now constructed of two flat-surfaced steel blocks 2.9 cm. long and separated by foils 0.15 mm. in thickness which left an opening

1 mm. long at the source end and 2 mm. long at the crystal end. The crystal was placed at a distance of 5 mm. from the end of the collimating system and the detecting plate was at a distance of 2.35 cm. from the crystal. The plate was slightly inclined to the direction of the beam so that the specular beam at grazing incidence was nearly normal to the plate. The angular aperture of the beam was 36′ and its trace on the plate subtended an angle of 1° 6′ from the crystal.

A typical exposure with this arrangement enlarged 2.86× is shown in Fig. 1. The reflection in this case took place at 3° 13′ from grazing incidence, and the reflected pattern appears in the center. For reference the incident beam was given a short exposure (which appears at the left) by removing the crystal from the beam. The primary spectra would lie outside of the borders of this photograph.

FIG. 1.

The secondary spectrum from LiF at 2° from grazing incidence.

The secondary spectrum appears in the photograph as the diffuse band running out from the specular beam and, contrary to hopes, it is still unseparated from the specular beam. From (1), the grating space of the secondary structure of LiF, for which $d = 4.0$ A., is $D \sim 70$ A. The first order spectrum of the wave-length distribution should therefore, have its maximum, according to the grating formula $\cos \theta_0 - \cos \theta = D$, at $\theta = 10°40'$, and this corresponds to a position in the figure 8.9 mm. from the center of the specular beam. This is marked by the black lines.

If the surface were crossed by uniformly spaced grating lines of equal reflecting power with the theoretical separation $D = 70$ A., a well-defined minimum should appear between the first order and the specular beam corresponding to the paucity of the Maxwellian distribution in the region of the short wave-lengths. In fact, this minimum should be resolved to the ex-

tent of appearing visible in the photograph if the grating distance D were so great as to separate the maximum of the first order by only three times the width of the specular beam. On the assumption of a uniform secondary surface structure, the failure to observe such a minimum leads to an angle of less than 6° 31′ for the first order diffraction of the most probable wave-length at the 3° 13′ angle of incidence which was used, and the corresponding grating space would have to be greater than 225A. The results of the experiment, therefore, are against the existence of a set of uniformly spaced segments of equal reflecting power, at separations of the order of magnitude predicted by theory.

This result, however, should not be regarded as a contradiction to Zwicky's theory. In fact, the existence of diffuse secondary spectra having well-defined directions is definite evidence for the existence of variations in the reflecting power of the surface in the expected directions parallel to the cleavage planes, and the failure to obtain an intensity distribution in these spectra corresponding to that which would be expected on the assumption of a uniform lattice, may be fully accounted for by nonuniformities either in the period of the density variations or in the reflecting power of the intervening surfaces. Such nonuniformities are to be expected on an artificially cleaved surface, and they do not have to be great to account for the results. Rayleigh [10] has pointed out that an average error of one fourth of the grating space in the positions of the lines of a grating is sufficient to weaken the first order spectrum and it is easy to see that such an error would carry with it an incomplete interference in the region between the orders which would produce the greatest diffuse intensity in the vicinity of the specular beam. A somewhat similar effect as regards the diffuse diffraction would also result from a grating, the lines of which were uniformly spaced but were of non-uniform reflecting power. Even though the secondary structure in the interior of the crystal were uniform in spacing, the etched surface would not necessarily present a set of groves of uniform width. Nonuniformities of this nature were, in fact, in evidence in the micro-photographs

[10] See E. C. Baly, "Spectroscopy," Vol. I, p. 159, Longmans Green and Co., 1929.

of the Bi surfaces studied by Goetz, and, although the blocks which he observed were integral multiples of a definite unit block, there was a considerable variation both in the number of units making up the surfaces which were developed by etching and in the extent to which these surfaces were developed. These nonuniformities were probably produced by the cleaving and etching processes, and it is possible that similar nonuniformities were produced in the case of the LiF crystal, even though a far higher degree of uniformity may have existed in a secondary block structure throughout the volume of the crystal.

SUMMARY.

When atom waves were reflected from a cleaved [100] surface of a LiF crystal, diffuse secondary spectra appeared which were well-defined in direction but were otherwise unresolved. This is regarded as evidence for a secondary structure on the surface of the crystal the directions of which are those of the cleavage planes. This secondary structure may have the period given by the theory of Zwicky but if so it must be slightly irregular either as regards this period or as regards the reflecting power of the intervening surfaces.

The writer wishes to acknowledge the assistance of Mr. M. A. Roesch in making the experiments.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

SPEED TESTS OF PIANO ACTIONS.

A manufacturer of pianos recently submitted to the Bureau of Standards models of three types of piano action requesting that speed tests be made. The models were provided with a mechanical device by means of which a rapid succession of strokes could be given the key. The rapidity of the action was recorded electrically on the oscillograph.

It was found that an action can be constructed which will respond as rapidly as 1600 strokes per minute, a speed much beyond that attainable by any artist.

Measurements were made of the speed of the hammer just before striking the string, which was found to be of the order of 3 feet per second. This is consistent with the high frefrequency because of the short stroke. In like manner the speed of the tip of a vibrating tuning fork is of the order of only one inch a second.

DECOMPOSITION OF TRICALCIUM SILICATE IN THE TEMPERATURE RANGE 1000–1300° C.

It has long been known that Portland cement, if heated to 1000° C. or higher, often deteriorates in its hydraulic properties. This is due to the liberation of lime, which if present to any great extent in the uncombined state causes unsoundness. Earlier studies have shown that the lime is liberated from the tricalcium silicate, which is one of the major constituents of Portland cement.

In the course of the present study it was found that at temperatures between 1000 and 1300° C., tricalcium silicate decomposes quite readily. The rate of decomposition is about fifteen times as great at 1175°, its maximum point, as it is at 1000°. Above 1175° the rate decreases rapidly, and at 1300° the decomposition is negligible. The presence of either of the products of the decomposition, dicalcium silicate and

* Communicated by the Director.

free lime, accelerates the reaction. For this reason, tricalcium silicate which has been exposed to moist air decomposes much more readily than that which has been freshly prepared, since the moisture of the air reacts with the silicate, liberating lime. The presence of gypsum also accelerates the decomposition probably because the gypsum dissociates and liberates lime.

PRESERVATION OF CORNERSTONE RECORDS.

It is the usual custom, as a part of the laying of the cornerstone of a public edifice, to deposit within the stone, with impressive ceremony, various records for the information of posterity. Unfortunately, posterity sometimes finds itself cheated by deterioration of the records that might have been prevented had adequate precautions been taken. A case of this kind occurred in Washington, D. C., recently, when records from the cornerstone of the razed Poli's theatre were examined. Although the records had been stored only 47 years, they were for the most part indecipherable. The records had been placed loose in a zinc box which was not air-tight, and the niche containing the box opened upward with no provision for drainage of water from it. Apparently water draining into the niche and remaining there had caused the rapid deterioration of the records as they were found to be very wet.

With this example of what might happen before them, those in charge of the erection of the District of Columbia's War Memorial recently obtained the collaboration of the bureau in safeguarding the cornerstone records, which was particularly important in this case as the records included the names of the 26,048 men of the District who served in the armed forces during the World War. The names were typewritten on paper made in the bureau paper mill, with every precaution to ensure permanence, solely from cotton fiber which is noted for its permanence. The records were sealed in an air-tight copper box from which the air had been removed and replaced with nitrogen which is relatively inert. The niche containing the box was so arranged that water would not drain into it.

While these precautions seem quite adequate, they are dwarfed by those recently used in Japan to preserve the names

of the victims of the earthquake of 1923. These precautions included the preparation of a special paper by the Japanese Government Paper Mill, wrapping of the records with oiled-silk and asbestos, the use of air-tight fused quartz containers in which the air had been replaced with argon gas as a preservative, and wrapping of the containers with asbestos, which was finally covered with carborundum. These precautions against chemical deteriorating agents, fire and physical stresses are destined to preserve the records for at least 10,000 years.

PERMANENCE QUALITIES OF BOOK PAPERS.

Tests of current commercial book papers have been made at the bureau relative to their value for the preservation of valuable records. The properties studied were their strength, chemical purity, both cellulosic and non-cellulosic, and their stability under an accelerated aging test made by heating them. As with other types of record paper studied, their quality for permanent records was found to be dependent on their strength and purity, neither of which bore a close relation to the kind or quality of the fibrous raw materials employed in their manufacture. In general, these papers were found to have poorer quality for permanent record use than those found in writing papers, and this is attributed to the more severe treatment given the fibers in obtaining the required printing qualities.

Based upon a classification system previously devised for record papers and on the test results obtained for the book papers, a suggested classification of papers for use in publications has been formulated. This details qualities considered desirable in papers for all types of printed records, permanent to temporary.

A detailed report of this study is contained in the September issue of the *Bureau of Standards Journal of Research.*

ULTRAVIOLET RADIOMETRY.

During the past month there has been assembled for publication data on the sun and on artificial light sources, obtained by means of the balanced thermocouple and filter method of ultraviolet radiometry.

An important accomplishment in this work was the completion of a series of tests of the physiological effect (erythema) produced by using measured amounts of spectrally pure ("monochromatic") and of spectrally heterogeneous ultraviolet radiation. The object of the measurements was to test this radiometric filter method as a dosage meter.

The sources used for producing the erythemic reaction differed very widely in ultraviolet spectral energy distribution. These sources were: (*a*) the sun, in which there is practically no radiation of wave-lengths shorter than 290 millimicrons; (*b*) a low voltage, relatively high vapor pressure, mercury arc, in a quartz bulb, which has a group of intense emission lines extending from 248 to 313 millimicrons; and (*c*) a high voltage, low vapor pressure, mercury arc in a quartz tube, in which 93 to 96 per cent. of the radiation of wave-lengths less than and including 313 millimicrons is emitted by the resonance line at 254 millimicrons.

Exposures were made on two observers. The observed time of exposure, and the calculated time, based upon the energy measured with the balanced filter radiometer and the spectral erythemic response curve, were found in close agreement—(within 5 to 8 per cent. in the case of the sun and the high pressure mercury arc). From this it appears that the spectral erythemic response curve is fairly well determined and that the heterogeneous ultraviolet radiation from various sources can be measured with considerable precision by means of the balanced filter radiometer.

NEW STANDARD SAMPLES NOW AVAILABLE.

The following new standard samples have recently been added to the list of Bureau of Standards standard samples.

The bureau issues at the present time some 100 standard samples comprising analyzed irons, steels, ferroalloys, nonferrous materials, ores, refractories, clays, etc., also pure chemicals and materials suitable for calibrating calorimetric bombs, polarimeters, thermometers, pyrometers, and other instruments. A complete list of these standards, prices, and shipping directions are given in Supplement to Circular No. 25, which can be obtained free on application to the bureau.

Sample No.	Name.	Constituents Determined.	Weight, Grams.	Price.
91	Opal glass	Complete analysis.	45	$2.00
92	Low boron glass	B_2O_3	45	2.00
93	Pyrex glass	Complete analysis.	45	2.00
94	Zinc base die casting alloy.	do	100	2.00
95	do	do	100	2.00
96	do	do	100	2.00
97	Flint clay	do	60	2.00
98	Plastic clay	do	60	2.00
99	Soda feldspar	do	40	2.00
100	Medium manganese steel.	C, Mn, P, S, Si, Cu, Ni, Cr, V, Mo.	150	2.00
101	18–8 Chrome-nickel steel (KA2S).	C, Mn, P, S, Si, Cu, Ni, Cr, V, Mo.	150	3.00

Orders for any of the above samples should be accompanied by money order or check made payable to the "Bureau of Standards, Department of Commerce." Detailed certificates of analysis are sent to the same destination as the samples.

WEAR RESISTANCE OF CARBON STEELS.

One of the major factors which causes deterioration of metals in service is wear. The cost of worn-out parts, the labor of replacement, and the time lost while repairing machinery represent a tremendous economic loss.

The selection, on the basis of laboratory tests, of those metals or alloys which best resist wear is extremely difficult because of the many complex factors involved. A study was recently completed of one special phase of this work, and a report will be issued in the September number of the *Bureau of Standards Journal of Research*. Tests of the wear-resistance of plain carbon steel were made in the Amsler machine (combined rolling and sliding friction under heavy pressures) to determine the effect of carbon content and heat treatment upon the wear-resistance of these particular steels. The results showed that the higher-carbon and the harder steels were more resistant to the particular type of wear studied than the lower-carbon and the softer steels.

MAGNETIC CHANGES IN STEEL DURING TEMPERING.

Various physical properties of steel are improved by heat treating processes. One of the most important of these is quenching from temperatures above the critical range into various cooling media, usually water or oil, and tempering or reheating to some temperature below this range. This latter temperature is usually determined by the composition of the steel and the use to which it is to be put. Quenching of steels of sufficient carbon content makes the material very hard and brittle, while tempering reduces the hardness to some extent and leaves the steel in a more ductile condition.

The tempering process is, therefore, of great importance in the heat treatment of steel. A study of this process in steel containing 0.75 per cent. carbon has recently been completed at the bureau, using thermomagnetic analysis as a method for studying the changes which take place in the tempering range. Thermomagnetic analysis, or the study of magnetic changes upon the application of heat, reveals constitutional changes in the material which may be difficult to follow by other methods of analysis, and its application to the study of progressive changes in the tempering range has proved to be of some value.

The magnetic changes taking place in the quenched steel upon heating and while holding at certain constant temperatures are quite clearly revealed. These magnetic changes represent changes in constituents or in the structure of the material itself. The tempering of quenched steel is shown to be progressive dependent upon the tempering temperature and the time at that temperature.

This investigation will be described in the September number of the *Bureau of Standards Journal of Research*.

THE FIRST SPECTRA OF KRYPTON AND XENON.

The stronger arc lines of krypton and xenon have been examined for hyperfine structures and a complete report will be published in the September number of the *Bureau of Standards Journal of Research*. Methods of observation include the use of a Hilger Fabry-Perot interferometer having quartz plates of 6 cm. aperature, a number of fixed etalons, and two

quartz Lummer-Gehrcke plates. The relative advantages of these two types of interferometers are considered. The results obtained by different methods show very satisfactory agreement. Structures of the following lines have been measured:

Krypton, 5570.2890, 7685.2472, 8059.5053, 8104.3660, 8281.042, and 8508.8736 A;

Xenon, 4193.5296, 4500.9772, 4734.1524, 8231.6348, 8409.190, and 8819.412 A.

The observed complex lines in the two spectra are not spectroscopically analogous. No definite numerical regularities in the spacing of components, such as have been reported in the case of neon, have been observed. Five of the six complex xenon lines are due to combinations with the low $^3P_2$ level. The regularities in the fine structures of neon are discussed. The satellites of neon lines are supposed to be due to Ne isotope 22. Krypton and xenon are reported to have 6 and 9 isotopes respectively, and the fine structure might be expected to be more complicated.

The relative abundance of isotopes of odd atomic weight in krypton and xenon leads to the consideration of the possibility that the satellites sufficiently separated for observation may be due to the nuclear moment of these odd isotopes, in accordance with a suggestion advanced by Schüler to explain the hyperfine structures in mercury. It is hoped that, by operating the sources at low temperatures, and using still higher resolving power, additional structures may be observed.

VIBRATIONS OF QUARTZ PLATES.

A large number of modes of vibration of 0° and 30° cut circular and rectangular crystalline quartz plates were studied experimentally at the bureau and a report will be issued in the September number of the *Journal of Research*. These plates are piezoelectric and are used extensively in radio technique as frequency standards. Various methods were employed in the work. The behavior of lycopodium powder when applied during vibration proved to be the best method for obtaining information. Many photographs of figures or

patterns obtained in this manner are shown in the report. Although of limited utility, two mechanical devices were employed with success. With one of these, nodes of radial displacement were detected at the periphery of circular plates vibrating at extensional nodes.

By means of the second device the direction of maximum radial displacement was determined for the two active lower frequency modes of a zero-degree circular plate. These two directions were found to correspond closely to the directions of the maximum and the minimum of Young's modulus. Simplified equations for Young's modulus, the modulus of rigidity, and Poisson's ratio, are derived for crystalline quartz, and graphs of the three functions given.

Pertinent facts were disclosed concerning the vibrations of specially oriented rectangular plates and rods. Modes of the former, when so oriented that their greatest dimension is parallel to a direction of a critical value of Young's modulus, were found to be of a relatively simple nature. Experimental values of Young's modulus based on the extensional vibration frequency of rods oriented with lengths parallel to computed directions of critical values of Young's modulus were found to agree closely with the corresponding computed values. By means of three rods cut with lengths in slightly different directions but all nearly parallel to the computed direction of the minimum of Young's modulus, this computed direction was roughly verified.

Certain methods of rigidly mounting quartz plates and rods which may be advantageously incorporated in the design of radio-frequency standards are indicated.

THE FRANKLIN INSTITUTE.

MEMBERSHIP NOTES.

ELECTIONS TO MEMBERSHIP.

(Stated Meeting Board of Managers, September 16, 1931.)

RESIDENT LIFE.

MR. LAWRENCE J. MORRIS, Merchant and Banker, 437 Chestnut Street, Philadelphia, Pa.

RESIDENT.

MR. EDWARD DAWSON, Design Draftsman, c/o Sperry Gyroscope Company, Brooklyn, New York. For mailing: 6111 McCallum Street, Germantown, Philadelphia, Pa.

DR. WAYNE B. NOTTINGHAM, Physicist, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

NON-RESIDENT.

MR. CHARLES O. DUEVEL, JR., Chief Engineer, American Thermos Bottle Company, Norwich, Connecticut.

MR. ERLE G. HILL, Metallurgist, Lukens Steel Company, Coatesville, Pa.

MR. PENROSE R. HOOPES, Consulting Mechanical Engineer, 43 Farmington Avenue, Hartford, Connecticut.

STUDENT.

MR. CARL EDWARD ATKINS, Radio Engineer, Sparks-Withington Company, Jackson, Michigan. For mailing: 302 East Biddle Street, Jackson, Michigan.

CHANGES OF ADDRESS.

MR. C. H. BIERBAUM, 113 Florence Avenue, Buffalo, New York.

MR. WILLIAM L. BROWN, 3D, 1140 Broad Street Station Building, Philadelphia, Pa.

MR. ARMIN ELMENDORF, 4414 West Division Street, Chicago, Illinois.

MR. W. G. H. FINCH, 315 Riverside Drive, New York City, New York.

MR. WALTON FORSTALL, JR., Rosemont, Pa.

MR. MAYO D. HERSEY, Evergreen Hall Apartments, Woodbury, New Jersey.

DR. S. S. KISTLER, 201 West Vermont Avenue, Urbana, Illinois.

LT. COL. CHESTER LICHTENBERG, 4616 South Wayne Avenue, Fort Wayne, Indiana.

MR. J. KENNETH W. MACALPINE, 196 North Oraton Parkway, East Orange, New Jersey.

MR. WILLIAM S. MASON, Box 266, Evanston, Illinois.

PROFESSOR J. C. MCLENNAN, 88 Prince Arthur Avenue, Toronto, Canada.

REAR ADMIRAL WILLIAM S. SIMS, U. S. N., 104 Marlboro Street, Boston, Massachusetts.

MR. CHARLES S. WURTS, JR., 1327 Walnut Street, Philadelphia, Pa.

LIBRARY NOTES.

RECENT ADDITIONS.

American Institute of Electrical Engineers. Transactions. Volume 50, No. 2. June, 1931.

American Medical Association. New and Nonofficial Remedies, 1929, 1930 and 1931. Three volumes.

ASHCROFT, E. W. Faraday. 1931.

ATTWOOD, EDWARD L. Theoretical Naval Architecture. Revised by Herbert S. Pengelly. New edition. 1931.

Barnwell Addresses. Volume I. 1922–1930. Edited with an introduction by John Louis Haney. 1931

BASSETT, HENRY NORMAN. The Chemical Technology of Steam-Raising Plant. 1931.

British Museum. A Guide to the Exhibition in the King's Library Illustrating the History of Printing, Music-Printing and Bookbinding. 1913.

BRUHAT, G. Cours d'Optique à l'Usage de l'Enseignement Supérieur. 1931.

Centenaire de Marcelin Berthelot 1827–1927. 1929.

CIERVA, JUAN DE LA, and DON ROSE. Wings of Tomorrow: the Story of the Autogiro. 1931.

DALBY, W. E. Power and the Internal Combustion Engine. 1931.

DIRAC, P. A. M. The Principles of Quantum Mechanics. 1930.

FRY, HARRY SHIPLEY. The Electronic Conception of Valence and the Constitution of Benzene. 1921.

GRANT, JULIUS. The Measurement of Hydrogen Ion Concentration. 1930.

GREGORY, JOSHUA C. The Scientific Achievements of Sir Humphry Davy. 1930.

HOLT, ANNE. A Life of Joseph Priestley. 1931.

Jahresbericht über die Leistungen der chemischen Technologie für das Jahr 1930, 2 abteilung. organischer Teil. 1931.

KENDREW, W. G. Climate: a Treatise on the Principles of Weather and Climate. 1930.

LEVY, S. I. The Rare Earths: their Occurrence, Chemistry and Technology. Second edition. 1924.

MELLOR, J. W. A Comprehensive Treatise on Inorganic and Theoretical Chemistry. Volume XI. 1931.

Merck's Index: an Encyclopedia for the Chemist, Pharmacist and Physician. Fourth edition. 1930.

MILLER, MAX C. Principles of Knitting. First edition. 1931.

MOORE, H. F., AND J. B. KOMMERS. The Fatigue of Metals. With chapters on the Fatigue of Wood and of Concrete. First edition. 1927.

Munich, deutsches Museum von Meisterwerken der Naturwissenschaften und Technik. Amtlicher Führer durch die Sammlungen. Zweite, vollständig neu bearbeitete und erweiterte Auflage. 1928.

ROOSES, MAX. Catalogue of the Plantin-Moretus Museum. Third English edition. 1921.

RUSSELL, PHILLIPS. Benjamin Franklin, the First Civilized American. 1926.

SMART, W. M. The Sun, the Stars and the Universe. 1928.

STROOBANT, P., AND OTHERS. Les Observatoires Astronomiques et les Astronomes. 1931.

United States Congressional Directory. 71st Congress, 3d Session. 1931.

WILLIAMS, ROGER J. An Introduction to Organic Chemistry. Second edition. 1931.

WINSHIP, GEORGE PARKER. Gutenberg to Plantin: an Outline of the Early History of Printing. 1926.

World Power Conference, Second, Berlin, 1930. Transactions, Volume 20, Index. No date.

BOOK REVIEWS.

A LIFE OF JOSEPH PRIESTLEY, by Anne Holt, with an introduction by Francis W. Hirst, xviii–221 pages, 12mo. London, Oxford University Press, Humphrey Milford, 1931. Price, $3.50.

Joseph Priestley, first of all a theologian, but one who also merits the titles of preacher, teacher, philosopher, and chemist. A contemporary of Adam Smith, Edmund Burke, Franklin, Washington and Jefferson, he has not been accorded so great renown as these worthies. The author, Anne Holt, has performed a very creditable piece of work in furnishing us with this well-balanced story of Priestley's life. A justifiable criticism might be that Miss Holt saw fit to limit the size of her book to a mere fraction of what it could have been, had all the interesting material at her disposal been utilized in its entirety.

Without a doubt, the Introduction, written by Francis W. Hirst, outranks any review which might be written and it would have been a far, far better stunt to have transposed that into this space allotted in the reviewing column.

Although Priestley's rôle as a chemist has been very adequately recounted by Sir Edward Thorpe for English Men of Science and in America by Edgar Fahs Smith, "Priestley in America," until now no one has made a conscientious attempt to fully portray the many-sided life and versatility of the man. Although he spent all but the last ten years of his life in England he had in that period become both noted and notorious more for his religious views than for his scientific accomplishments. In America almost the reverse was the case. Upon his arrival in 1794 via the steamer "Sansom" he was hailed as the discoverer of oxygen, suggested for the presidency of the American Philosophical Society, and invited to fill the chair of Chemistry at the University of Pennsylvania. Despite such honors, it is doubtful that they offset his disappointment at not being offered a pulpit. The story of Priestley's last years contains many elements of sadness which were caused through bereavements of those nearest to him, the inevitable physical disablements of the human body nearing the end of its course and the fear that political and religious chauvinists would succeed in doing him harm. Further light upon the man's mental make-up and character is afforded by his defense of the Phlogiston Theory at a period when its last vestiges of support had been forever vanquished by experimental fact and irrefutable logic.

He died February 6, 1804. Today, in Northumberland, Pa., stands a shrine which houses much of his chemical apparatus, scientific instruments and books; a few yards away is the structure which served as his home; a few minutes ride

will bring one to the cemetery where lies a grave and stone bearing the name Joseph Priestly and the epitaph:

> "Return into thy rest, O my soul, for the Lord hath dealt bountifully with thee. I will lay me down in peace and sleep till I wake in the morning of the resurrection."

T. K. CLEVELAND.

FARADAY. 113 pages, 8vo. London, The British Electrical and Allied Manufacturers Association. 1931.

Six years ago the British Society of Chemical Industry commemorated the centenary of Michael Faraday's discovery of benzene. This year the Royal Institution assisted by the Institution of Electrical Engineers, have planned a series of Faraday Centenary Celebrations in recognition of this great scientist's discovery of electro-magnetic induction. In this discovery lay the origin of the dynamo.

This book on the life and work of Michael Faraday has geen published by the British Electrical and Allied Manufacturers Association and written by E. W. Ashcroft. Considerable pains have been taken in its printing and the person responsible for the typography deserves praise. The pages are uncut.

Michael Faraday, born September 22, 1791, was the son of a blacksmith. His earlier years fell within a period of hard times for the laboring classes, and at the age of nine the young Michael had learned the secret of making one loaf of bread last a week. Although circumstances would permit only the most meager of educations, consisting of not much more than the three "R's," for one of Michael Faraday's temperament and abilities his first job probably turned out to be the best possible substitute for college. At the age of thirteen he was apprenticed to a bookseller.

Most of us have heard how Faraday was given tickets for a series of lectures delivered by Sir Humphrey Davy at the Royal Institution; how he made careful and complete notes on these lectures and how these notes, submitted with his application, won for him the position of laboratory assistant under Davy at the Royal Institution.

So, at the age of twenty-two, Faraday's feet had been directed to paths of scientific adventure and his face turned toward the unexplored and shadowy vistas which lay in the realm of natural philosophy. An account of his experiences, experiments and discoveries in the years to follow comprises one of the most fascinating stories of scientific adventure ever written. Let us in retrospection accompany Faraday on his European trip which he took with Davy shortly after joining the Institution. In Paris, Davy is shown a new substance which gives off a deep violet vapor when heated. It is iodine, which Davy shows to be an element. In Genoa, they experiment upon electric fish to see whether they can decompose water when connected in a manner similar to voltaic battery. In Florence, at the Accademia del Cimento, they borrow the large "burning glass" and manage to burn diamond in a glass globe filled with oxygen. Back at home Faraday begins a long series of accomplishments first in chemistry and later in physics, his discoveries in which have had such far reaching influence in the development of the Age of Electricity. He analyzes a specimen

of lime from an Italian volcano, extends the application of the oxy-hydrogen blowpipe, liquefies chlorine, discovers and describes several chlorinated hydrocarbons and, perhaps most important of all, separates benzene from oil gas and determines its constitution. In 1831 he stumbled upon his great discovery "that an electric current is induced in a wire helix during the alteration in the strength of a magnetic force." No need to enlarge upon its significance—we all recognize the important place dynamos occupy in present-day affairs. To Faraday must be attributed the discovery of the laws of electrolytic dissociation, the concept of magnetic tubes of force, the origination of the children's Christmas lectures at the Royal Institution, the magnetic rotation of polarized light planes, the property of diamagnetism.

Faraday was particularly fortunate to have begun his scientific career under the famous Sir Humphrey Davy, who is probably best known for his invention of the miner's safety lamp. To Davy, Faraday proved a worthy successor. Against his unabating zeal for scientific activity, nature began to exact toll. A period of intense activity and research would be suddenly terminated by spells of dizziness, headaches and loss of memory. In 1866 the spirit was for the last time curbed by the now flickering flames of bodily fires and on August 25, 1867, he died in his chair, in his study.

T. K. CLEVELAND.

ESSENTIALS OF QUANTITATIVE CHEMICAL ANALYSIS. A Laboratory Manual for Colleges, by Wilfred W. Scott, Sc.D., Professor of Chemistry, University of Southern California. viii–219 pages, 8vo. Easton, The Chemical Publishing Co., 1931. Price $2.75.

It is not difficult to comprehend the reasons which created a necessity for a second edition of this manual. As the title announces, the author, Wilfred W. Scott, has chosen the most essential procedures in quantitative analysis and carefully demonstrates their application to practical problems most frequently encountered by the chemist in his work.

Like most books on quantitative analysis, this starts with a description of gravimetric methods followed by those for volumetric. The gravimetric section consists of chapters each one describing the methods for determining some particular element or radicle. In addition to the methods of procedure, the author takes pains to explain the reasons for such and concludes the chapter with a paragraph on their practical applications. Among the gravimetric methods described we mention at random those for the commercial evaluation of bauxite for alumina, the determination of sulfur in iron pyrites, sulfur in steel, dolomite and limestone, constitution of brass, mercury in its ores.

The volumetric section begins with a description of the general procedure and includes such practical methods as those for determining phosphorus in rocks, bone ash, fertilizers, iron and steel; manganese in ores, ferromanganese and steel alloys; calcium in water, lead and zinc in their respective ores.

The microchemical method for determining arsenic will have special appeal for those interested in forensic chemistry. Under colorimetric methods we find described the determination of ammonia in water with Nessler's reagent. A separate chapter has been allotted to a discussion of pH, its practical determination and its importance in pathological studies.

The author is to be commended for having included methods worked out by specialists in their particular fields. We refer to such as—Rapid methods for water analysis, iron and steel analyses, coke analysis. At the end of the book the instructor will find a list of questions which may be assigned to his students. Inside the rear cover is a table of solubilities of the more common inorganic substances.

T. K. CLEVELAND.

SEEING, A PARTNERSHIP OF LIGHTING AND VISION. By M. Luckiesh, D.Sc., Director, Lighting Research Laboratory, General Electric Company, Nela Park, Cleveland, and Frank K. Moss, E.E., Lighting Research Laboratory, Nela Park, Cleveland. vii–241 pages plus 26 pages, plates, illustrations, tables, 8vo. Baltimore, The Williams and Wilkins Co., 1931. Price $5.00.

With "Seeing," the senior author M. Luckiesh, collaborating with F. K. Moss, adds another book to the constantly growing list testifying to his authorship. "Seeing, a partnership of lighting and vision" has, in its entirety, been woefully neglected. True, much work has been done on the subject of vision, and in contrast, relatively little upon light and lighting. As a result, the authors have directed their energies toward a series of researches which they hope will form a foundation for the science of seeing. Their findings and conclusions should prove immensely valuable to lighting specialists and others interested directly or indirectly in all that is desirable in methods of proper illumination. Let us glance over the chapters and see—by virtue of visible acuity, illumination and contrast—what of general interest may lie there.

A Half-Seeing World.—The evolution of man took place under an illumination of 1000 foot candles or more. Many persons now work under illumination intensities of two to three foot candles. Nearsightedness is commonly caused by an inadequate intensity of illumination for seeing reading matter.

Fatigue.—Very difficult to determine or to evaluate. Too many uncontrollable factors both psychological and physiological. Why is it many of us can attack a problem more clearheadedly at the end of a long tiresome day than in the morning when our awaking faculties are regained afresh?

The Science of Seeing.—To measure seeing such factors as size, contrast, brightness and time of exposure must be rigidly controlled and their variations accurately measured.

Visual Acuity.—"Normal" vision is regarded as the ability to see details of about one minute visual angle. Physically, such a detail represents an object 0.0041 inch in size and viewed from a distance of 14 inches. Minimum size of an object depends in part upon contrast and intensity of illumination. If eyesight is defective, correct glasses aid enormously.

Contrast.—This is the relation between the brightness of the object and that of its background. The jeweller attains contrast by means of a black velvet background.

Brightness.—Meet the millilambert unit of brightness. Consider your telephone directory, it is not so bright when compared to this book. Reflection factor of its paper is 57 compared to 80. Per cent. contrast is 80 compared to 97. For equal visibility its brightness must be 100 per cent. greater.

Speed of Seeing.—Therein lies the magician's success, also, that of the movies. Under ideal conditions the average necessary time of exposure was 0.15 second.

The Pupil of the Eye.—Normal size of pupil is between 3 and 5 millimeters. When out-doors looking at overcast sky, pupil was 1.6 mm. diameter. This increased to 5.3 mm. after five minutes in a dark room.

Adaptation.—Roughly it requires as many minutes for the pupil to open to its maximum diameter as it does seconds to close to its minimum. We become aware of this delay in adaptation when approaching automobile headlights temporarily blind us.

Effect of Surroundings.—Color has its place if rightly chosen, but imagine one trying to work effectively in a machine shop painted a boudoir pink and wearing cerise overalls and jumper.

Glare and Visibility.—In spite of considerable effort, automobile headlight glare has not been completely eliminated. One way highways or strong highway illumination are suggested as solutions.

Spectral Character of Light.—Just why did the human eye choose to be sensitive to electro-magnetic wave-lengths ranging between 3900 and 7700 Angstrom units? One of the authors is to be credited with the pioneer development of the well known daylight lamp. Why the blue-bulb headlight lamps are not better than the others.

Subnormal Vision.—Modern optometry recognizes several thousand varying degrees of defective vision which may be corrected. Hyperopia is the rule with infants. The tables showing correlation between defective vision and age, type of occupation, progress in school, and hourly earnings of workers are very interesting.

Characteristics of Objects.—Shadows exert many peculiar effects. Ever notice how much bumpier the highway looks when illuminated by headlights? The advantages of highlights for finding a needle on a non-contrasting background.

Foot Candles.—The inadequacy of the present lighting code and a more modern one soon to be adopted.

General Lighting Plus.—Advantages of adding to general lighting carefully designed localized lighting properly controlled. They are many.

Psychological Aspects.—Beneficent influences exerted through the choice of well-lighted and appropriate colored working environment. Factors which are receiving widespread recognition today and are being rapidly adopted.

Cost and Value.—A problem involving consideration of the law of diminishing returns. One must determine the point beyond which increased illumination would prove uneconomical. The average family spends about seven cents a day for electric light in the home. If an 1876 laborer had used tallow candles to give as much light as is in the home today, working twenty-four hours a day would no more than pay for them.

In the back of the book will be found a number of plates which act as illustrations for portions of the text. In addition the authors have added a supplement whereby the reader may conduct his own visual test experiments.

T. K. CLEVELAND.

THE GREEN LEAF. By D. T. MacDougal, M.S., Ph.D., LL.D., Research Associate, Carnegie Institution of Washington. 142 pages, 12mo. New York, D. Appleton & Co., 1930. Price $2.00.

The Green Leaf tells lucidly of those basic processes whereby trees grow and chlorophyll converts sap into sugar under the energy of light. MacDougal has

written a clear story of facts so simple that they still escape the understanding of intense research. How the tiny chloroplasts perform is omitted from the engaging story of plant life for the well known reason that no one yet knows this secret. A great deal is told, however, of the mechanics of sap movement, the opening and shutting of the stomata that line the under leaf and the arrangement of cells in the green leaf. No one, at least in New England, can escape the blaze of autumn coloring. Very few realize "the disintegration of chlorophyll that yields the yellow xanthophyll, and also unmasks the other yellowish-red carotinoids . . . the concentration of sugar in the inactive leaves is favorable to the formation of anthocyans, red and blue colors, while the action of oxidases, as seen on the cut surface of ripe apples, produces blacks and browns."

Upon last analysis all life leans on the chloroplast. The Green Leaf expands one's gratitude at the same time stretching one's understanding.

H. W. ELKINTON.

LES QUANTA. Par Georges Déjardin, Professeur a la Faculté des Sciences de Lyon. 224 pages, illustrations, 16mo. Paris, Librairie Armand Colin, 1930. Price ten francs 50.

Not so many months ago the timely subject of "*Ondes et Electrons*" by Pierre Bricout was added to the *Collection Armand Colin*. Now very fittingly comes a companion volume on the allied subject *Quanta*, which in the hands of Professor Planck applied to black-body radiation thirty years ago astonished the scientific world and latterly has bidden fair to remake it.

The book is intended for a first reading and general view of the origin, development and application of quanta in various branches of physics. To that end, the author has eliminated the difficult calculations which are not indispensable in order to find space to include a discussion of hypotheses and experimental proofs. Despite its small compass, it is a very complete review sustained by a satisfying complement of mathematical deduction.

LUCIEN E. PICOLET.

LA TELEPHONIE. Par Robert Dreyfus, Ingenieur des Postes et Telegraphes 191 pages, illustrations, 16mo. Paris, Librairie Armand Colin, 1931. Price ten francs 50.

An adequate explanation of the operation of modern telephone installations is scarcely possible without recourse to the language of the electrical engineer. This condensed account on wire telephony is obviously addressed to readers who, if they are not already familiar with the methods of dealing with oscillatory currents, have the prerequisite knowledge to follow the development of the differential equations which the author has found it necessary to employ in the consideration of transmission lines. There is, however, not a great amount of this mathematical material and the reader who has a general knowledge of electrical appliances can profitably follow the descriptive matter on modern practice in telephony. Manual and automatic systems are discussed and the principles of carrier-wave methods are noted. The book is a highly informative condensed account of the principles of modern telephony over wires.

LUCIEN E. PICOLET.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 382. Elastic Instability of Members having Sections Common in Aircraft Construction, by George W. Trayer and H. W. March, 42 pages, illustrations, quarto. Washington, Government Printing Office, 1931. Price twenty-five cents.

Two fundamental problems of elastic stability are discussed in this report, which was prepared by the Forest Products Laboratory for publication by the National Advisory Committee for Aeronautics. In Part I formulas are given for calculating the critical stress at which a thin, outstanding flange of a compression member will either wrinkle into several waves or form into a single half wave and twist the member about its longitudinal axis. A mathematical study of the problem, which together with experimental work has led to these formulas, is given in an appendix. Results of tests substantiating the recommended formulas are also presented. In Part II the lateral buckling of beams is discussed. The results of a number of mathematical studies of this phenomenon have been published prior to this writing, but very little experimentally determined information relating to the problem has been available heretofore. Experimental verification of the mathematical deductions is supplied in this report.

Report No. 383. On the Theory of Wing Sections with Particular Reference to the Lift Distribution—by Theodore Theodorsen, 16 pages, illustrations, quarto. Washington, Government Printing Office, 1931. Price ten cents.

This paper gives a simple and exact method of calculating the lift distribution on thin wing sections. The most essential feature of the new theory is the introduction of an "ideal angle of attack," this angle being defined as that at which the flow enters the leading edge smoothly or, more precisely, as the angle of attack at which the lift at the leading edge equals zero. The lift distribution at this particular angle is shown to be a characteristic property of the section and has been termed the "basic distribution." It is shown that the lift of a wing section may always be considered to consist of (*a*) the basic distribution and (*b*) the additional distribution where the latter is independent of the mean camber line and thus identical for all thin sections. The specific reason for the poor aerodynamic qualities of thin wing sections is pointed out as being due to the fact that the additional lift in potential flow becomes infinite at the leading edge.

The theory is in consequence adapted to describe some of the properties of actual wing sections. It is established that the essential parameter occurring in this analysis is the radius of curvature at the leading edge. The location and magnitude of the maximum lift intensity is determined. It is further pointed out that the actual slope of the lift curve is dependent on this parameter.

The theoretical lift distribution is compared with the distribution obtained by direct measurement on a number of the more conventional wing sections. The results check satisfactorily and may be considered as a confirmation of the validity of the theory.

The new theory will be of value in the further improvements of airplane wings. It is pointed out that airplanes should be operated near the ideal angle of attack. The theory will also be of use in calculating the structural strength of wing sections.

Report No. 385. Wind Tunnel Tests on Airfoil Boundary Layer Control using a Backward-Opening Slot, by Millard J. Bamber, 38 pages, illustrations, quarto. Washington, Government Printing Office, 1931. Price twenty cents.

This report presents the results of an investigation to determine the effect of boundary layer control on the lift and drag of an airfoil. Boundary layer control was accomplished by means of a backward-opening slot in the upper surface of the hollow airfoil. Air was caused to flow through this slot by a pressure which was maintained inside the airfoil by a blower. Various slot locations, slot openings, and wing pressures were used. The tests were conducted in the 5-foot atmospheric wind tunnel of the Langley Memorial Aeronautical Laboratory.

The quantity of air flowing through the slot per unit time was measured and is presented in coefficient form. A coefficient is derived from which the power required to maintain the air flow through the slot may be computed.

The effect of each variable is illustrated by characteristic curves. A discussion indicating the advantages which might be possible by the application of boundary layer control to an airplane is included.

A discussion of the various forces produced on the airfoil by this type of boundary layer control and their resultants is given in Appendix I.

Under the test conditions, the maximum lift coefficient was increased about 96 per cent. for one slot arrangement, and the minimum drag coefficient was decreased about 27 per cent. for another, both being compared with the results obtained with the unslotted airfoil. It is believed from the results of this investigation that the above effects may be increased by the use of larger slot openings, better slot locations, multiple slots, improved airfoil profiles, and trailing edge flaps.

Report No. 390. The Effect of Valve Timing upon the Performance of a Supercharged Engine at Altitude and an Unsupercharged Engine at Sea Level, by Oscar W. Schey and Arnold E. Biermann, 13 pages, illustrations, quarto. Washington, Government Printing Office, 1931. Price ten cents.

This investigation was conducted to determine the comparative effects of valve timing on the performance of an unsupercharged engine at sea level and a supercharged engine at altitude. The tests were conducted by the staff of the National Advisory Committee for Aeronautics on the N. A. C. A. Universal test engine. The timing of the four valve events was varied over a wide range; the engine speeds were varied between 1,050 and 1,500 r.p.m.; the compression ratios were varied between 4.35 : 1 and 7.35 : 1. The conditions of exhaust pressure and carburetor pressure of a supercharged engine were simulated for altitudes between 0 and 18,000 feet. The results show that optimum valve timing for a supercharged engine at an altitude of 18,000 feet differs slightly from that for an unsupercharged engine at sea level. A small increase in power is obtained by using the optimum timing for 18,000 feet for altitudes above 5,000 feet. The timing of the intake opening and exhaust closing becomes more critical as the compression ratio is increased.

Report No. 392. Reduction of Turbulence in Wind Tunnels, by Hugh L. Dryden, 11 pages, illustrations, quarto. Washington, Government Printing Office, 1931. Price ten cents.

A brief nonmathematical outline is given of modern views as to the nature of the effect of turbulence, and their bearing on the desirability of designing wind tunnels for small or large turbulence. Experiments made on a particular wind tunnel for the purpose of reducing the turbulence are described, to illustrate the influence of certain factors on the magnitude of the turbulence. Moderate changes in the size, shape, and wall thickness of cells of the honeycomb were found to have little effect. The addition of a room honeycomb at the entrance was also of little value in reducing the turbulence. The turbulence decreased with increasing distance between the honeycomb and the measuring station. A further decrease was obtained by using a large area reduction in the entrance cone with the honeycomb at the extreme entrance end.

The measurements of turbulence were made by the use of spheres and also by the use of the hot wire anemometer as described in reference 5. The present work was conducted with the coöperation and financial assistance of the National Advisory Committee for Aeronautics.

PUBLICATIONS RECEIVED.

Dielectric Constant and Molecular Structure, by Charles Phelps Smyth, 214 pages, illustrations, 8vo. New York, The Chemical Catalog Co., 1931. Price $4.00.

Verhandlungen des 3ten Internationalen Kongresses für Technische Mechanik, Stockholm, 24–29 August 1930, three volumes, illustrations, quarto, Stockholm, Sveriges Litografiska Tryckerier. Price 130 kr.

Economic Control of Manufactured Product, by W. A. Shewhart, 501 pages, tables, diagrams, 8vo. New York, D. Van Nostrand Company, Inc., 1931. Price $6.50.

Science in Action, a sketch of the value of scientific research in American industries, by Edward R. Weidlein and William A. Hamor, 310 pages, illustrations, 8vo. New York and London, McGraw-Hill Book Company, Inc., 1931. Price $3.00.

Union Geodesique et Geophysique Internationale, Section de Magnetisme et Electricite Terrestres, Comptes Rendus de l'Assemblee de Stockholm 15–23 Aout 1930, publies par les soins de Ch. Maurain, 477 pages, illustrations, plates, 8vo. Paris, Les Presses Universitaires de France, 1931.

Pratt Institute Free Library, Technical Books of 1930, A selection, compiled by Donald Hendry, Twenty-third issue, 28 pages, 16mo. Brooklyn, Pratt Institute, 1931.

U. S. Bureau of Mines, Potash in 1930, by A. T. Coons, 8 pages, tables, 8vo. Washington, Government Printing Office, 1931. Price five cents. Technical Paper 499, Treating a Complex Ore. Data from experimental work on ores in the Denver Laboratories of the Complex Ores Recovery Co., by G. L. Oldright, 101 pages, tables, 8vo. Washington, Government Printing Office, 1931. Price fifteen cents.

U. S. Coast and Geodetic Survey, Special Publication No. 173, Latitude Redeterminations, by Frederic W. Darling, 17 pages, tables, plates, 8vo. Washington, Government Printing Office, 1931. Price ten cents.

A Yearbook of Railroad Information, 1931 edition, 94 pages, diagrams, 16mo. New York, Committee on Public Relations of the Eastern Railroads.

U. S. Bureau of Standards, Research Paper No. 321. Volume Changes in Brick Masonry Materials, by L. A. Palmer, 23 pages, diagrams, tables, 8vo. Washington, Government Printing Office, Price ten cents. Research Paper No. 326. The Freezing Point of Platinum, by Wm. F. Roeser, F. R. Caldwell, H. T. Wensel, ten pages, diagrams, 8vo. Washington, Government Printing Office, Price five cents.

Bell Telephone Laboratories, Monographs: No. B-579, Power System Communication, by C. A. Booker and M. E. Clark, 16 pages, illustrations, maps, 8vo. B-580, Communication Services of Electric Utilities, by R. N. Conwell, G. M. Keenan, C. F. Craig, E. C. Briggs, 20 pages, illustrations, 8vo., New York, 1931.

The Magnesite Deposits of Mytileni, by V. Jerome Akylas, reprinted from The Mining Magazine, July 1931, 7 pages, illustrations, 8vo. London, Mining Publications, Ltd.

Massachusetts Institute of Technology, Abstracts of Scientific and Technical Publications, Number 8, July 1931, 81 pages, 8vo. Cambridge, The Technology Press.

National Advisory Committee for Aeronautics, Technical Notes: No. 386, Effect of Nose Shape on the Characteristics of Symmetrical Airfoils, by Robert M. Pinkerton, Langley Memorial Aeronautical Laboratory, 16 pages, tables, diagrams, quarto, Washington, Committee, 1931. No. 387, The Pressure Distribution Over a Modified Elliptical Wing Tip on a Biplane in Flight, by Richard V. Rhode and Eugene E. Lundquist, Langley Memorial Aeronautical Laboratory, 14 pages, tables, diagrams, quarto. Washington, Committee, 1931. No. 388. A Comparison of the Aerodynamic Characteristics of Three Normal and Three Reflexed Airfoils in the Variable Density Wind Tunnel, by George L. Defoe, Langley Memorial Aeronautical Laboratory, 12 pages, tables, diagrams, quarto. Washington, Committee, 1931. No. 389, The N. A. C. A. Apparatus for Studying the Formation and Combustion of Fuel Sprays and the Results from Preliminary Tests, by A. M. Rothrock, Langley Memorial Aeronautical Laboratory, 22 pages, plates, quarto. Washington, Committee 1931. No. 390, A Method for Reducing the Temperature of Exhaust Manifolds—by Oscar W. Schey and Alfred W. Young, Langley Memorial Aeronautical Laboratory, 9 pages, plates, quarto. Washington, Committee, 1931. No. 391. Tests of N. A. C. A. Airfoils in the Variable-Density Wind Tunnel, Series 43 and 63, by Eastman N. Jacobs and Robert M. Pinkerton, Langley Memorial Aeronautical Laboratory, 32 pages, tables, plates, quarto. Washington, Committee, 1931.

Carte Geologique du Massif du Mont-Blanc, Partie Francaise, par MM. Paul Corbin et Nicolas Oulianoff, Le Tour avec Notice Explicative, 16 pages, plates, 8vo. Paris, Librairie G. Jacquart, 1931. Price 20 francs.

CURRENT TOPICS.

Preservative Treatment of Wood Shows Slight Decline in 1930 —(*U. S. Department of Agriculture Press Service*). An 8 per cent. decline in the quantity of wood given preservative treatment last year as compared with 1929 was reported by the U. S. Forest Service to-day in announcing its annual compilation of statistics on the quantity of wood treated and the preservatives used.

The following statement was issued by the Branch of Research of the Forest Service:

"Final tabulations have been completed for incorporation in the twenty-second statistical report which the Forest Service compiles annually in coöperation with the American Wood Preservers' Association. The report covers the activities of the industry for the calendar year 1930. These reports for previous years have almost invariably shown increases in both preservatives used and material treated. The business depression of 1930 no doubt affected this industry, as it did many others, although perhaps to a smaller degree. It is to be expected, therefore, that reductions from the figures shown in the report for 1929, the peak year of the industry, would be registered both in the quantity of preservatives used and in the material treated in 1930.

"In 1930 a total of 204 plants were in active operation, or one more than reported in 1929. Four new plants were constructed in the year, or three less than were built in 1929. Of the 204 plants that were active in the year covered by this report, 134 were of the pressure cylinder type, 53 were non-pressure (open tank) plants, and 17 were equipped for both pressure and non-pressure treatment.

"The total quantity of wood treated in 1930 was 332,318,577 cubic feet, 8 per cent. less than the quantity reported in 1929. All but 10 per cent. of the quantity of material reported in 1930 was treated by pressure processes.

"All but one of the eight classes of material treated in 1930 showed decreases from the quantities reported in 1929. The exception was switch ties, which showed a slight increase. The greatest decrease was registered by cross-ties, amounting to 23,267,988 cubic feet. The next largest decrease occurred in poles treated, the quantity reported in 1930 being less by 1,896,171 cubic feet than in 1929. Wood blocks ranked third with a decrease of 1,839,685 cubic feet, and construction timbers was fourth with a decrease of 1,190,442 cubic feet. Next in point of quantity was miscella-

neous material with a decrease of 935,429 cubic feet. Slight decreases were also recorded in the quantity of crossarms and piles treated.

"In 1930 the quantity of creosote used by the wood-preserving industry was 213,904,421 gallons, or 5.9 per cent. less than was reported in 1929. The amount of petroleum consumed by the industry was less in 1930 than in 1929 by 1,555,865 gallons.

"The 1930 consumption of zinc chloride by the treating plants of this country was less than that reported the year before by nearly 30 per cent. This reduction in the consumption of zinc chloride results in large part, perhaps, from the lessened use of this salt alone or in mixture with creosote in the treatment of cross-ties and switch ties. Consumption of zinc chloride in the treatment of other classes of material in 1930 showed an increase over the quantity so used in 1929.

"The total quantity of miscellaneous preservatives used by the wood preserving industry in 1930 consisted of 1,770,925 pounds of salts and 202,891 gallons of liquids. The salts comprised sodium fluoride, sodium arsenite, arsenic trioxide, zinc-meta-arsenite, sodium chloride, mercuric chloride and Wolman salts. The liquids included Ac-zol, Montan wax and Carbosota."

R.

THE FRANKLIN INSTITUTE

OFFICERS FOR 1931

President	NATHAN HAYWARD
Vice-presidents	HENRY HOWSON C. C. TUTWILER WALTON FORSTALL W. CHATTIN WETHERILL
Secretary	HOWARD MCCLENAHAN
Assistant Secretary and Librarian	ALFRED RIGLING
Treasurer	BENJAMIN FRANKLIN
Assistant Treasurer	MARSHALL S. MORGAN
Controller	WILLIAM F. JACKSON, JR.

Board of Managers

GEORGE H. BENZON, JR.
EDWARD G. BUDD
FRANCIS T. CHAMBERS
G. H. CLAMER
WALTON CLARK
CYRUS H. K. CURTIS
SAMUEL S. FELS
ARTHUR W. GOODSPEED
CLARENCE A. HALL
GEORGE A. HOADLEY
A. ATWATER KENT
L. H. KINNARD
ELISHA LEE
ROBERT W. LESLEY
LIONEL F. LEVY
MALCOLM LLOYD, JR.
ROBERT MCLEAN
CHARLES PENROSE
GEORGE WHARTON PEPPER
JAMES S. ROGERS
GEORGE D. ROSENGARTEN
HASELTINE SMITH